CHAPTER V

SECONDARY DIFFRACTION

In the previous chapters, an approximate solution of diffraction problems was carried out which was based on the representation of the fringing field in the form of the sum of the fields from the uniform and nonuniform parts of the surface current. The first field was found by quadratures, and the second field by approximation; it was assumed that the nonuniform part of the current near the discontinuity (edge) of a surface is the same as on a corresponding wedge.

However, the fields found by such a method are actually the fields from the currents flowing, not only on the flat and curved parts of the body's surface, but also to some extent on the geometric extension of these sections. The error in the expressions for the fringing field which is thus introduced is most significant with a glancing incident wave, when the edge zone occupied by the nonuniform part of the current is noticeably broadened, and also with a glancing radiation, when the direction to the observation point forms a small angle with the given section of the surface. In these cases, the results obtained earlier are in need of substantial corrections. We already talked about this briefly in § 6 and § 12.

For the purpose of refining the solutions which were found previously, it is necessary to assume that in actuality the currents flow only on the body's surface, and that a wave travelling from one edge to the other will undergo a perturbation at the latter. The process of forming the fringing field when this occurs may be investigate in the following way. The edge wave propagated from one of the edges is diffracted by the other edges; the waves arising with this in turn are diffracted by adjacent edges, etc. In this chapter, we will investigate the case when the dimensions of the surface faces are so large in comparison with the wavelength that it is sufficient to limit oneself to considering the diffraction of only the primary edge waves. This phenomenon we shall call secondary diffraction.

In this chapter, secondary diffraction by an infinitely long strip (§ 20 - § 23) and by a circular disk (§ 24) is studied. The solution of these problems may be obtained by means of the principle of duality from the solution of the diffraction problems for an infinite slit and a circular hole in a flat, ideally conducting screen In the latter case, the physical treatment of diffraction of edge waves is significantly simpler; it is exactly for this reason, therefc that almost all diffraction studies of edge waves are related to holes in a plane screen. However, we will not take such a path, but we shall investigate a strip and a disk directly. This approach has the advantage that it is easily generalized to the case of three-dimensional bodies.

§ 20. Secondary Diffraction by a Strip. Formulation of the Problem.

Let an infinitely thin, ideally conducting strip of width 2a and unlimited length be orientated in space as shown in Figure 45. A plane electromagnetic wave incident normal to the strip's edges is directed at an angle α to the plane xoz and has the following form:

$$\mathbf{E}=\mathbf{E}_0 e^{ik(x\cos\alpha+y\sin\alpha)},\quad \mathbf{H}=\mathbf{H}_0 e^{ik(x\cos\alpha+y\sin\alpha)}. \qquad (20.01)$$

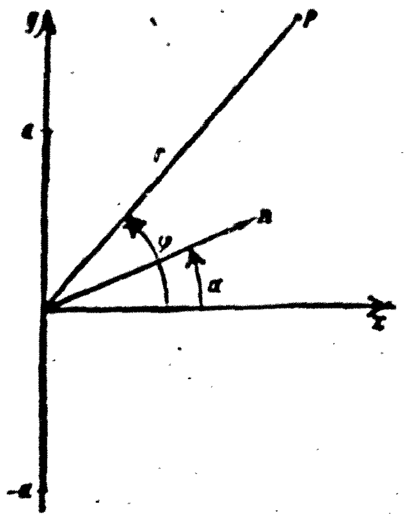

Figure 45. The transverse cross section of a strip with the plane xoy, x = 0, y = a and x = 0, y = -a are the coordinates of the strip's edge; n is the normal to the incident plane wave front.

In § 6 approximation expressions were found for the fringing field in the far zone which did not consider the interaction of the edges. In the case of E-polarization of the incident wave $(E_0 \parallel 0z)$, these expressions may be represented in the form

$$\left.\begin{aligned} E_z = -H_\varphi = E_{0z}\left[f(1)\,e^{ika(\sin\alpha - \sin\varphi)} + \right. \\ \left. + f(2)\,e^{-ika(\sin\alpha - \sin\varphi)}\right]\frac{e^{i\left(kr + \frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}, \\ E_\varphi = H_z = 0, \end{aligned}\right\} \tag{20.02}$$

and in the case of H-polarization $(H_0 \parallel 0z)$

$$\left.\begin{aligned} H_z = E_\varphi = H_{0z}\left[g(1)\,e^{ika(\sin\alpha - \sin\varphi)} + \right. \\ \left. + g(2)\,e^{-ika(\sin\alpha - \sin\varphi)}\right]\frac{e^{i\left(kr + \frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}, \\ H_\varphi = E_z = 0. \end{aligned}\right\} \tag{20.03}$$

Let us recall that the functions f and g included here are determined in the region $|\varphi| < \frac{\pi}{2}$ (when $|\alpha| < \frac{\pi}{2}$) by the following relationships:

$$f(1) = \frac{\cos\frac{\alpha+\varphi}{2} - \sin\frac{\alpha-\varphi}{2}}{\sin\alpha - \sin\varphi}, \quad f(2) = -\frac{\cos\frac{\alpha+\varphi}{2} + \sin\frac{\alpha-\varphi}{2}}{\sin\alpha - \sin\varphi}, \tag{20.04}$$

$$g(1) = -f(2), \quad g(2) = -f(1). \tag{20.05}$$

The first terms in Equations (20.02) and (20.03) describe cylindrical waves diverging from edge 1 (y = a), and the second terms describe the cylindrical waves diverging from edge 2 (y = -a). The nonuniform part of the current on each side of the strip also has the form of waves which diverge from edges 1 and 2, and are an "analytical extension" of the corresponding terms in Equations (20.02) and (20.03) The current wave encountering the opposite edge is reflected from it. Or else one may say that each of the cylindrical waves propagated from edge 1 or 2 undergoes diffraction by the opposite edge (secondary diffraction).

If the strip's width is sufficiently large in comparison with the wavelength, then one may approximately assume that the oncoming current wave near the strip's edge will be the same as on a corresponding half-plane excited by a linear source, the moment of which is selected in a definite way. It is also obvious that the current waves reflected from the edge will also coincide. Consequently, the problem of secondary diffraction by a strip may be reduced to the problem of the diffraction of a cylindrical wave by a half-plane.

The field created at the point P by a current filament parallel to the half-plane's edge and passing through the point Q (Figure 46) may be found by means of the reciprocity principle. In the case of E-polarization, it is determined by the relationship

$$E_z = \frac{p_z}{p_{0z}} E_z(Q), \tag{20.06}$$

and in the case of H-polarization

$$H_z = \frac{m_z}{m_{0z}} H_z(Q). \tag{20.07}$$

Here p_z (m_z) is the electric (magnetic) moment of the current filament passing through the Q; p_{oz} (m_{oz}) is the moment of the auxiliary current filament passing through the point P with the coordinates (ϕ'', R), and H_z (Q) or E_z (Q) is the field created by the auxiliary filament at the point Q.

Now let us remove the auxiliary current filament to such a distance that the cylindrical wave arriving from it may be considered

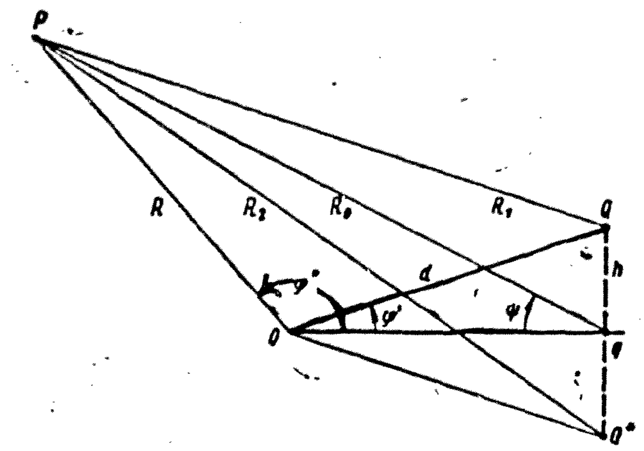

Figure 46. Diffraction of a cylindrical wave by a half-plane. Q is the source, Q* is the mirror image of the source, and P is the observation point.

to be a plane wave on the section from the edge of the half-plane to the point Q. In this case, in accordance with § 1 and § 2 the field created by it at the point Q will equal

$$\left.\begin{aligned} E_z(Q) &= E_{0z}(0)\,[u(d, \varphi' - \varphi'') - u(d, \varphi' + \varphi'')], \\ H_z(Q) &= H_{0z}(0)\,[u(d, \varphi' - \varphi'') + u(d, \varphi' + \varphi'')]. \end{aligned}\right\} \tag{20.08}$$

The functions u introduced here are determined (for the values $0 \leqslant \varphi'' \leqslant \pi$) by the equations

$$\left.\begin{aligned} u(d, \varphi' - \varphi'') &= e^{-ikd\cos(\varphi' - \varphi'')}\frac{e^{-i\frac{\pi}{4}}}{\sqrt{\pi}} \times \int\limits_{\infty\cos\frac{\varphi'-\varphi''}{2}}^{\sqrt{2kd}\cos\frac{\varphi'-\varphi''}{2}} e^{iq^2}dq + \\ &+ \begin{cases} e^{-ikd\cos(\varphi'-\varphi'')} & \text{with } 0 \leqslant \varphi' \leqslant \pi + \varphi'' \\ 0 & \text{with } \pi + \varphi'' \leqslant \varphi' \leqslant 2\pi, \end{cases} \\ u(d, \varphi' + \varphi'') &= e^{-ikd\cos(\varphi' + \varphi'')}\frac{e^{-i\frac{\pi}{4}}}{\sqrt{\pi}} \times \int\limits_{\infty\cos\frac{\varphi'+\varphi''}{2}}^{\sqrt{2kd}\cos\frac{\varphi'+\varphi''}{2}} e^{iq^2}dq + \\ &+ \begin{cases} e^{-ikd\cos(\varphi'+\varphi'')} & \text{with } 0 \leqslant \varphi' \leqslant \pi - \varphi'' \\ 0 & \text{with } \pi - \varphi'' \leqslant \varphi' \leqslant 2\pi, \end{cases} \end{aligned}\right\} \tag{20.09}$$

and the quantities E_z (0) and H_z (0) are the values of the primary field created by the auxiliary filament at points corresponding to the half-plane's edge. In accordance with Equations (1.21) and (1.22) this field may be represented when kR >> 1 in the form

$$\left.\begin{aligned} E_{0z}(0) &= ik^2 p_{0z}\sqrt{\frac{2\pi}{kR}}\, e^{i\left(kR - \frac{\pi}{4}\right)}, \\ H_{0z}(0) &= ik^2 m_{0z}\sqrt{\frac{2\pi}{kR}}\, e^{i\left(kR - \frac{\pi}{4}\right)}. \end{aligned}\right\} \tag{20.10}$$

Consequently, an electric current filament located above an ideally conducting half-plane excites, at the point P, the field

$$E_z = ik^2 p_z [u(d, \varphi' - \varphi'') - u(d, \varphi' + \varphi'')] \sqrt{\frac{2\pi}{kR}} e^{i\left(kR - \frac{\pi}{4}\right)}, \qquad (20.11)$$

and a magnetic current filament excites, at the point P, the field

$$H_z = ik^2 m_z [u(d, \varphi' - \varphi'') + u(d, \varphi' + \varphi'')] \sqrt{\frac{2\pi}{kR}} e^{i\left(kR - \frac{\pi}{4}\right)}. \qquad (20.12)$$

It is easy to see that the exponent $e^{ik[R - d\cos(\varphi' - \varphi'')]}$ in these expressions corresponds to the primary cylindrical wave arriving at the observation point P, and the exponent $e^{ik[R - d\cos(\varphi' + \varphi'')]}$ corresponds to the reflected cylindrical wave.

The moments m_z and p_z must be selected in such a way that in the direction $\phi'' = \pi$ (Figure 46) the filament would create a field equal to the field of the primary edge wave above an infinite, ideally conducting plane. We will conclude these calculations in the following sections, but for now let us make still one other comment on the formulation of the problem.

In the previous chapters it was shown that the scattering object may be approximated by a series of sources — "luminous" lines and points. Therefore, the problem of secondary diffraction may be formulated as a problem of searching for functions which describe the continuous change of the field of each such source during the passage through the boundary of the light and shadow corresponding to the source.

§ 21. Secondary Diffraction by a Strip (H-Polarization)

A current filament with the moment m_z which is positioned above an ideally conducting plane ($h = 0$, Figure 46) creates in space the field

$$H_z = ik^2 m_z 2\pi H_0^{(1)}(kR_1). \tag{21.01}$$

Far from the filament (when $kR_1 \gg 1$), this field is described by the asymptotic expression

$$H_z = 4\pi k^2 m_z \frac{e^{i\left(kR_1 + \frac{\pi}{4}\right)}}{\sqrt{2\pi kR_1}}. \tag{21.02}$$

But the primary edge wave in the direction $\phi'' = \pi$ takes the value

$$H_z = H_{oz}(Q)\, g(Q) \frac{e^{i\left(kR_1 + \frac{\pi}{4}\right)}}{\sqrt{2\pi kR_1}}, \tag{21.03}$$

where $H_{oz}(Q)$ is the field of the incident plane wave at the point Q; $g(Q)$ is the value of the angular function of the primary cylindrical wave in the direction towards the opposite edge of the strip. Equating Expressions (21.02) and (21.03), we find the filament's moment, the field of which we use to approximate the primary edge wave, in the form

$$m_z = \frac{1}{4\pi k^2} H_{oz}(Q)\, g(Q). \tag{21.04}$$

As a result, the field created by the filaments located above the half-plane $-a \leqslant y \leqslant \infty$ and corresponding to edge 1 (Figure 47) may be represented for region $|\varphi| \leqslant \frac{\pi}{2}$ in the form

$$H_z(1) = H_z^+(1) + H_z^-(1). \tag{21.05}$$

The function

$$H_z^+(1) = \frac{1}{\pi} H_{oz} g(1) \times \int_{\infty}^{2\sqrt{ka}\cos\left(\frac{\pi}{4} - \frac{\varphi}{2}\right)} e^{iq^2} dq \frac{e^{ikr}}{\sqrt{2kr}} e^{ika(\sin\alpha - \sin\varphi)} \tag{21.06}$$

describes the wave radiated by the source m_{1z}^+, and the fu ction

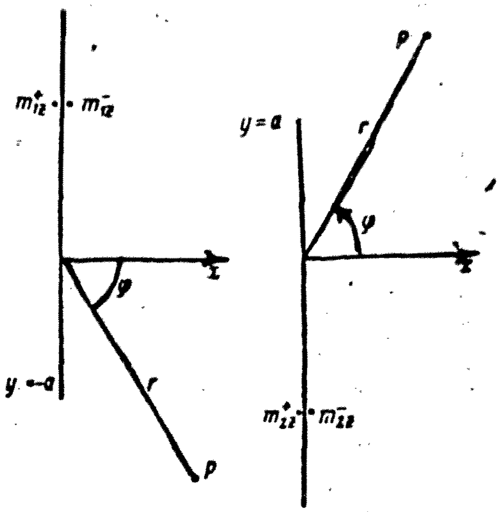

Figure 47. The problem of secondary diffraction by a strip.
m_{1z}^+ and m_{1z}^- are the sources, the fields of which are used when approximating the primary edge wave being propagated from edge 1 ($y = a$);
m_{2z}^+ and m_{2z}^- are the sources, the fields from which are used when approximating the primary wave from edge 2 ($y = -a$).

$$H_z^-(1) = \frac{1}{\pi} H_{0z}\bar{g}(1) \times \int_{\infty}^{2\sqrt{ka}\cos\left(\frac{\pi}{4}-\frac{\varphi}{2}\right)} e^{iq^2} dq \frac{e^{ikr}}{\sqrt{2kr}} e^{ika(\sin\alpha-\sin\varphi)} + H_{0z}\bar{g}(1)\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}} e^{ika(\sin\alpha-\sin\varphi)} \quad (21.07)$$

describes the wave radiated by the source m_{1z}^-. The sum of these waves equals

$$H_z(1) = \frac{2}{\pi} H_{0z}\bar{g}(1) \times \int_{\infty}^{2\sqrt{ka}\cos\left(\frac{\pi}{4}-\frac{\varphi}{2}\right)} e^{iq^2} dq \frac{e^{ikr}}{\sqrt{2kr}} e^{ika(\sin\alpha-\sin\varphi)} + H_{0z}\bar{g}(1)\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}} e^{ika(\sin\alpha-\sin\varphi)} \quad (21.08)$$

The first term in this expression is the desired secondary wave from edge 2, and the second term is the field radiated by the filament which is loacted above the ideally conducting plane $x = 0$ and has the moment

$$m_{1z}^- = \frac{1}{4\pi k^2} H_{0z}\bar{g}(1) e^{ika\sin\alpha}, \quad (21.09)$$

where

$$\bar{g}(1) = g(1)\Big|_{\varphi=-\frac{\pi}{2}+0}. \quad (21.10)$$

Summing the secondary wave which has been found with the unperturbed primary wave from edge 1, we obtain

$$H_z(1-2) \doteq$$

$$= H_{0z}\frac{2}{\pi}\bar{g}(1)\times\int_{\infty}^{2\sqrt{ka}\cos\left(\frac{\pi}{4}-\frac{\varphi}{2}\right)} e^{iq^2}dq\,\frac{e^{ikr}}{\sqrt{2kr}}e^{ika(\sin\alpha-\sin\varphi)}+$$

$$+H_{0z}g(1)\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}e^{ika(\sin\alpha-\sin\varphi)}. \qquad (21.11)$$

This expression reduces to zero if one assumes $\phi = -\pi/2$; consequently the secondary diffraction eliminates the field discontinuities which occurred in the previous approximation when $\phi = -\pi/2$. However, in the direction $\phi = \pi/2$ the field (21.11) is different from zero. Since H_z is an odd function of the x coordinate, the relationship $H_z|_{\varphi=\frac{\pi}{2}-0}\neq 0$ means that the fringing field components H_z and E_ϕ will undergo a discontinuity with a transition through the direction $\phi = \pi/2$. The reason for such a jump, as before, is that in our calculations the plane $x = 0$ is a plane of currents. By finding the secondary wave from edge 2, we actually considered that the diffraction takes place not on the edge of a finite width strip, but on the edge of an ideally conducting half-plane $-a\leqslant y\leqslant\infty$.

Again the resulting discontinuity has an order of magnitude of $\frac{1}{\sqrt{ka}\sqrt{kr}}$. It is clear that one may completely eliminate the field discontinuities only with consideration of multiple diffraction. However, the calculation of fields arising with multiple diffraction requires specific consideration of the following terms in order of smallness in the expansion of the primary edge in inverse powers of $\sqrt{kr}$ (see, for example, [46]). All this greatly complicates the calculations. Therefore, we, using the condition $ka \gg 1$, will limit ourselves to an investigation of secondary diffraction, and in order to eliminate the discontinuities in the plane $x = 0$, we will proceed in the following way.

Let us consider the quantity $\tilde{g}(1)$ in the Expression (21.11) to be a function of the angle ϕ [see Equation (20.05)], that is, let us replace $\tilde{g}(1)$ by the function $g(1)$. In this case the equation

$$H_z(1-2)=$$
$$=H_{0z}\cdot\frac{2}{\pi}g(1)\times\int\limits_{\infty}^{2\sqrt{ka}\cos\left(\frac{\pi}{4}-\frac{\varphi}{2}\right)} e^{iq^2}dq\,\frac{e^{ikr}}{\sqrt{2kr}}e^{ika(\sin\alpha-\sin\varphi)}+$$
$$+H_{0z}g(1)\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}e^{ika(\sin\alpha-\sin\varphi)}. \tag{21.12}$$

will give qualitatively correct results not only when $\varphi\approx-\frac{\pi}{2}$, but also with all other values of ϕ. Actually, the Fresnel integral is close to zero if $\sqrt{ka}\cos\left(\frac{\pi}{4}-\frac{\varphi}{2}\right)\gg 1$, and in Equation (21.12) only the second term remains, as must be the case. Therefore, Equation (21.12) may be investigated as an interpolation equation, and it may be applied with any values of $\varphi\left(|\varphi|\leqslant\frac{\pi}{2}\right)$. It is easy to establish that now the fringing field does not undergo a discontinuity with the passage through plane x = 0, since Expression (21.12) becomes zero when $\varphi=\pm\frac{\pi}{2}$.

It is interesting to note that Equation (21.12) automatically follows from Equation (21.08) if in the latter equation one replaces $\tilde{g}(1)$ by $g(1)$. Essentially, this substitution is equivalent to the assumption that the moments of the filaments, the fields of which are used for approximating the primary edge waves, depend on the radiation direction (that is, on the azimuth ϕ of the observation point)

$$m_{1z}^{-}=-m_{1z}^{+}=\frac{1}{4\pi k^2}H_{0z}g(1)e^{ika\sin\alpha}. \tag{21.13}$$

Such a determination of the moments of the auxiliary linear sources is used, for example, in the work of Millar [47].

Precisely in the same way that Equations (21.06) and (21.07) were obtained, we find (when x > 0)

$$H_z^+(2) = \frac{1}{\pi} H_{0z}\bar{g}(2) \int_{\infty}^{2\sqrt{ka}\cos\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)} e^{iq^2}dq \frac{e^{ikr}}{\sqrt{2kr}} e^{-ika(\sin\alpha-\sin\varphi)}, \tag{21.14}$$

$$H_z^-(2) = \frac{1}{\pi} H_{0z}\bar{g}(2) \int_{\infty}^{2\sqrt{ka}\cos\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)} e^{iq^2}dq \frac{e^{ikr}}{\sqrt{2kr}} e^{-ika(\sin\alpha-\sin\varphi)} +$$

$$+ H_{0z}\bar{g}(2) \frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}} e^{-ika(\sin\alpha-\sin\varphi)}. \tag{21.15}$$

These expressions give the field created by the filaments which are located above the ideally conducting half-plane $-\infty \leqslant y \leqslant a$ and have the moments

$$m_{2z}^- = -m_{2z}^+ = \frac{1}{4\pi k^2} H_{0z}\bar{g}(2) e^{-ika\sin\alpha}. \tag{21.16}$$

In accordance with Equation (21.04), here

$$\bar{g}(2) = g(2)\Big|_{\varphi=\frac{\pi}{2}-0}. \tag{21.17}$$

Furthermore, summing (21.14) and (21.15), we obtain

$$H_z(2) = H_{0z}\frac{2}{\pi}\bar{g}(2) \int_{\infty}^{2\sqrt{ka}\cos\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)} e^{iq^2}dq \frac{e^{ikr}}{\sqrt{2kr}} e^{-ika(\sin\alpha-\sin\varphi)} +$$

$$+ H_{0z}\bar{g}(2) \frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}} e^{-ika(\sin\alpha-\sin\varphi)}. \tag{21.18}$$

Here the first term is the desired secondary wave from edge 1, and the second term is the field radiated by the filament which is locat above the ideally conducting plane x = 0 and has the moment m_{2z}^-. Summing the secondary wave which has been found with the unperturbed primary wave from edge 2, we have

$$H_z(2-1)=H_{0z}\frac{2}{\pi}\tilde{g}(2)\int_{\infty}^{2\sqrt{ka}\cos\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)} e^{iq^2}dq\,\frac{e^{ikr}}{\sqrt{2kr}}e^{-ika(\sin\alpha-\sin\varphi)}+$$
$$+H_{0z}g(2)\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}e^{-ika(\sin\alpha-\sin\varphi)}. \qquad (21.19)$$

It is not difficult to see that the resulting expression becomes zero if one assumes $\varphi=\frac{\pi}{2}$ in it. Consequently, the secondary diffraction eliminates the field discontinuity which we had earlier (§ 6) when $\varphi=\frac{\pi}{2}$, but at the same time it leads to a field discontinuity when $\varphi=-\frac{\pi}{2}$. Again the resulting field discontinuity may be eliminated by the above indicated method, replacing the quantity $\tilde{g}(2)$ by g(2) — that is, by assuming the moments m_{2z}^{-} and m_{2z}^{+} depend on the observation angle ϕ. Actually as a result of such a substitution, we obtain from (21.19) the expression

$$H_z(2-1)=H_{0z}\frac{2}{\pi}g(2)\int_{\infty}^{2\sqrt{ka}\cos\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)} e^{iq^2}dq\,\frac{e^{ikr}}{\sqrt{2kr}}e^{-ika(\sin\alpha-\sin\varphi)}+$$
$$+H_{0z}g(2)\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}e^{-ika(\sin\alpha-\sin\varphi)}, \qquad (21.20)$$

which vanishes when $\varphi=\pm\frac{\pi}{2}$. This expression may be investigated as an interpolation equation which describes the field created in the region $|\varphi|<\frac{\pi}{2}$ by the primary wave of edge 2 with consideration of its diffraction at edge 1.

Now summing (21.12) and (21.20), we obtain the following expression for the total field scattered by the strip:

$$H_z=H_{0z}[G(1,\varphi)g(1)e^{ika(\sin\alpha-\sin\varphi)}+$$
$$+G(2,\varphi)g(2)e^{-ika(\sin\alpha-\sin\varphi)}]\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}. \qquad (21.21)$$

Here

$$G(1, \varphi) = \frac{2}{\sqrt{\pi}} e^{-i\frac{\pi}{4}} \times \int_0^{2\sqrt{ka}\cos\left(\frac{\pi}{4}-\frac{\varphi}{2}\right)} e^{iq^2} dq \tag{21.22}$$

is the shading function of the primary wave travelling from edge 1, and

$$G(2, \varphi) = \frac{2}{\sqrt{\pi}} e^{-i\frac{\pi}{4}} \times \int_0^{2\sqrt{ka}\cos\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)} e^{iq^2} dq \tag{21.23}$$

is the shading function of the primary wave travelling from edge 2. These functions show that the primary wave from edge 1 undergoes the greatest perturbation when $\varphi \approx -\frac{\pi}{2}$, and the wave from edge 2 undergoes the greatest perturbation when $\varphi \approx \frac{\pi}{2}$.

An important property of Equation (21.21) is that it becomes zero when $\varphi = \pm\frac{\pi}{2}$ — that is, the field discontinuity which we had earlier at the plane x = 0 is completely eliminated.

In concluding this section, let us return to Expressions (21.11) and (21.19) which lead to discontinuities of the fringing field in the plane of the strip (x = 0). One may show that the sum of these expressions

$$H_z = H_{0z} \frac{2}{\pi} \frac{e^{ikr}}{\sqrt{2kr}} \left[\bar{g}(1) \int_{\infty}^{2\sqrt{ka}\cos\left(\frac{\pi}{4}-\frac{\varphi}{2}\right)} e^{iq^2} dq\, e^{ika(\sin\alpha - \sin\varphi)} + \right.$$
$$\left. + \bar{g}(2) \int_{\infty}^{2\sqrt{ka}\cos\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)} e^{iq^2} dq\, e^{-ika(\sin\alpha - \sin\varphi)} \right] +$$
$$+ H_{0z} [g(1) e^{ika(\sin\alpha - \sin\varphi)} + g(2) e^{-ika(\sin\alpha - \sin\varphi)}] \frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}} \tag{21.24}$$

agrees, when $\sqrt{ka}\cos\left(\frac{\pi}{4}\pm\frac{\varphi}{2}\right)\gg 1$, with the asymptotic solution obtained in the book [50] by means of integral equations. The solution found in [50] has the greatest precision when $\alpha\approx 0, \varphi\approx 0$, and it is complete useless if $\alpha\approx\pm\frac{\pi}{2}$ or $\varphi\approx\pm\frac{\pi}{2}$.

§ 22. Secondary Diffraction by a Strip (E-Polarization)

It is known that a current filament with an electric moment p_z which is found at a distance h from an infinite, ideally conducting plane (see Figure 46) creates in space the field

$$E_z = ik^2 p_z \pi [H_0^{(1)}(kR_1) - H_0^{(1)}(kR_2)]. \tag{22.01}$$

With small values of h (and $R_{1,2} \gg kh^2$), this expression is transformed to the form

$$E_z = -i2p_z k^3 h \sin\psi \sqrt{\frac{2\pi}{kR_0}}\, e^{i\left(kR_0+\frac{\pi}{4}\right)}. \tag{22.02}$$

The primary edge wave is determined by the relationship

$$E_z = E_{0z}(q) f(q) \frac{e^{i\left(kR_0+\frac{\pi}{4}\right)}}{\sqrt{2\pi kR_0}}, \tag{22.03}$$

where $E_{0z}(q)$ is the value of the incident plane wave field at the point q ($R_0 = 0$). Consequently, the primary edge wave in the directi $\psi = 0$ may be investigated as the wave from a current filament located above an ideally conducting plane if one assumes the filament moment to be equal to

$$p_z = \frac{i}{4\pi k^3 h} E_{0z}(q) \frac{f(q)}{\sin\psi}\bigg|_{\psi=0}. \tag{22.04}$$

The field, created at the point P by the current filament with a moment p_z which is parallel to the half-plane's edge and passes through the point Q, is determined by Expression (20.11). Expanding the right-hand member of this expression into a series in terms of the small quantity h($h \to 0$) and limiting ourselves to the first term which is different from zero, we obtain

$$E_z = ik^2 p_z \frac{\partial}{\partial h} [u(d, \varphi' - \varphi'') - - u(d, \varphi' + \varphi'')]\Big|_{h=0}^{h} \sqrt{\frac{2\pi}{kR}}\, e^{i\left(kR - \frac{\pi}{4}\right)}. \quad (22.05)$$

By means of relationships (22.04) and (22.05), one may show that current filaments with moments p_{1z}^{-} and p_{1z}^{+} which are located on the ideally conducting half-plane $-a \leqslant y \leqslant \infty$ and correspond to edge 1 (see Figure 47) create in the region $|\varphi| \leqslant \frac{\pi}{2}$ the field

$$E_z = E_{0z} \frac{2}{\pi} \frac{e^{ikr}}{\sqrt{2kr}} \left[\cos\varphi \int_{\infty}^{2\sqrt{ka}\cos\left(\frac{\pi}{4} - \frac{\varphi}{2}\right)} e^{iq^2} dq + \right.$$
$$\left. + \frac{i}{2\sqrt{ka}} \sin\left(\frac{\pi}{4} - \frac{\varphi}{2}\right) e^{2ika(1+\sin\varphi)} \right] \bar{f}(1)\, e^{ika(\sin\alpha - \sin\varphi)} +$$
$$+ E_{0z} \bar{f}(1) \cos\varphi \frac{e^{i\left(kr + \frac{\pi}{4}\right)}}{\sqrt{2\pi kr}} e^{ika(\sin\alpha - \sin\varphi)}. \quad (22.06)$$

The current filaments with the moments p_{2z}^{-} and p_{2z}^{+} which are located above the ideally conducting half-plane $-\infty \leqslant y \leqslant a$ and correspond to edge 2 create in the same region the field

$$E_z = E_{0z} \frac{2}{\pi} \frac{e^{ikr}}{\sqrt{2kr}} \left[\cos\varphi \int_{\infty}^{2\sqrt{ka}\cos\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)} e^{iq^2} dq + \right.$$
$$\left. + \frac{i}{2\sqrt{ka}} \sin\left(\frac{\pi}{4} + \frac{\varphi}{2}\right) e^{2ika(1-\sin\varphi)} \right] \bar{f}(2)\, e^{-ika(\sin\alpha - \sin\varphi)} +$$
$$+ E_{0z} \bar{f}(2) \cos\varphi \frac{e^{i\left(kr + \frac{\pi}{4}\right)}}{\sqrt{2\pi kr}} e^{-ika(\sin\alpha - \sin\varphi)}. \quad (22.07)$$

The first terms in Expressions (22.06) and (22.07) are the desired secondary waves, and the last terms in the expressions are the fields from the current filaments located above the ideally conducting plane x = 0 and having the moments

$$p_{1z}^{-} = \frac{i}{4\pi k^2 h} E_{0z}\cdot \tilde{f}(1)\, e^{ika\sin\alpha}, \quad p_{2z}^{-} = \frac{i}{4\pi k^2 h} E_{0z}\cdot \tilde{f}(2)\, e^{-ika\sin\alpha}, \qquad (22.08)$$

where

$$\tilde{f}(1) = \frac{f(1)}{\cos\varphi}\bigg|_{\varphi=-\frac{\pi}{2}+0}; \quad \tilde{f}(2) = \frac{f(2)}{\cos\varphi}\bigg|_{\varphi=\frac{\pi}{2}-0}. \qquad (22.09)$$

Summing the secondary waves which have been found with the unperturbed primary waves, we obtain the total field scattered by the strip

$$\begin{aligned} E_z = -H_\varphi = E_{0z}\frac{2}{\pi}\frac{e^{ikr}}{\sqrt{2kr}}\Bigg\{\tilde{f}(1)\Bigg[\cos\varphi \times \int\limits_{\infty}^{2\sqrt{ka}\cos\left(\frac{\pi}{4}-\frac{\varphi}{2}\right)} e^{iq^2}dq + \\ + \frac{i}{2\sqrt{ka}}\sin\left(\frac{\pi}{4}-\frac{\varphi}{2}\right) e^{2ika(1+\sin\varphi)}\Bigg] e^{ika(\sin\alpha-\sin\varphi)} + \\ + \tilde{f}(2)\Bigg[\cos\varphi \times \int\limits_{\infty}^{2\sqrt{ka}\cos\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)} e^{iq^2}dq + \\ + \frac{i}{2\sqrt{ka}}\sin\left(\frac{\pi}{4}+\frac{\varphi}{2}\right) e^{2ika(1-\sin\varphi)}\Bigg] e^{-ika(\sin\alpha-\sin\varphi)}\Bigg\} + \\ + E_{0z}[f(1)\, e^{ika(\sin\alpha-\sin\varphi)} + f(2)\, e^{-ika(\sin\alpha-\sin\varphi)}] \times \\ \times \frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}. \quad (22 \end{aligned} \qquad (22.10)$$

Now assuming, as in the case of the H-polarization, that the moments $p_{1z}^{\pm}$ and $p_{2z}^{\pm}$ depend on the angle ϕ, by replacing

$$\tilde{f}(1) \text{ by } \frac{f(1)}{\cos\varphi} \text{ and } \tilde{f}(2) \text{ by } \frac{f(2)}{\cos\varphi}, \qquad (22.11)$$

we obtain

$$\begin{aligned} E_z = -H_\varphi = E_{0z}[F(1,\varphi)\, f(1)\, e^{ika(\sin\alpha-\sin\varphi)} + \\ + F(2,\varphi)\, f(2)\, e^{-ika(\sin\alpha-\sin\varphi)}]\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}, \end{aligned} \qquad (22.12)$$

where

$$F(1, \varphi) = \frac{2}{\sqrt{\pi}} e^{-i\frac{\pi}{4}} \left[\int_0^{2\sqrt{ka}\cos\left(\frac{\pi}{4}-\frac{\varphi}{2}\right)} e^{iq^2} dq + \frac{i}{4\sqrt{ka}} \frac{e^{2ika(1+\sin\varphi)}}{\cos\left(\frac{\pi}{4}-\frac{\varphi}{2}\right)} \right],$$

$$F(2, \varphi) = \frac{2}{\sqrt{\pi}} e^{-i\frac{\pi}{4}} \left[\int_0^{2\sqrt{ka}\cos\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)} e^{iq^2} dq + \frac{i}{4\sqrt{ka}} \frac{e^{2ika(1-\sin\varphi)}}{\cos\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)} \right] \qquad (22.13)$$

are the shading functions. They show that the primary wave from edge 1 undergoes the greatest perturbation near $\varphi = -\frac{\pi}{2}$, and the wave from edge 2 undergoes the greatest perturbation in the vicinity of $\varphi = \frac{\pi}{2}$.

§ 23. The Scattering Characteristics of a Plane Wave by a Strip

Expressions (21.21) and (22.12) which were obtained above for the field scattered by a strip approximately take into account the interaction of the edges and are valid when $|\varphi| < \frac{\pi}{2}$. However, they are not applicable with a glancing incidence of a plane wave on a strip (when $\alpha \approx \pm\frac{\pi}{2}$).

In order to find equations which are applicable in this case, let us proceed in the following way. Let us write the expressions for the field radiated by the strip in the direction α with the incidence of a plane wave in the direction ϕ (Figure 45)

$$E_z = -H_\varphi = E_{0z}[F(2, \alpha) f(1) e^{ika(\sin\alpha - \sin\varphi)} + F(1, \alpha) f(2) e^{-ika(\sin\alpha - \sin\varphi)}] \frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}},$$

$$H_z = E_\varphi = H_{0z}[G(2, \alpha) g(1) e^{ika(\sin\alpha - \sin\varphi)} + G(1, \alpha) g(2) e^{-ika(\sin\alpha - \sin\varphi)}] \frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}.$$

re $|\alpha| \leq \frac{\pi}{2}$, but ϕ cannot approximate $-\pi/2$. Now let us note that the expressions for the fringing field must satisfy the reciprocity principle — that is, they must not change with the simultaneous replacement of α by ϕ and ϕ by α. Comparing Equations (21.21), (20.12) and (23.01), it is not difficult to obtain the expressions

$$\left.\begin{aligned} E_z = -H_\varphi = E_{0z}[F(1,\varphi)F(2,\alpha)f(1)e^{ika(\sin\alpha-\sin\varphi)} + \\ + F(2,\varphi)F(1,\alpha)f(2)e^{-ika(\sin\alpha-\sin\varphi)}]\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}, \\ H_z = E_\varphi = H_{0z}[G(1,\varphi)G(2,\alpha)g(1)e^{ika(\sin\alpha-\sin\varphi)} + \\ + G(2,\varphi)G(1,\alpha)g(2)e^{-ika(\sin\alpha-\sin\varphi)}]\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}, \end{aligned}\right\} \qquad (23.02)$$

which satisfy the reciprocity principle, have no discontinuities anywhere, and are suitable for making calculations with any values of α and φ $\left(|\alpha| \leq \frac{\pi}{2}, |\varphi| \leq \frac{\pi}{2}\right)$. From the second equation of (23.02), it follows that $H_z = E_\phi = 0$ when $\varphi = \pm\frac{\pi}{2}$ — that is, the fringing field does not experience discontinuities in the plane x = 0. Moreover, $H_z = E_\phi = 0$ with any values of ϕ if $\alpha = \pm\frac{\pi}{2}$ — that is, a plane wave polarized perpendicularly to the strip does not undergo diffraction with a glancing incidence.

The resulting Equations (23.02) may be investigated as interpolation equations. Actually, with $|\alpha| \ll \frac{\pi}{2}$ when $\sqrt{ka}\cos\left(\frac{\pi}{4} \pm \frac{\alpha}{2}\right) \gg 1$ the functions $F(1,\alpha)$, $F(2,\alpha)$, $G(1,\alpha)$ and G $(2, \alpha)$ are close to one, and Equations (23.02) change into the previous Expressions (21.21) and (22.12). But if $|\varphi| \ll \frac{\pi}{2}$ and $\sqrt{ka}\cos\left(\frac{\pi}{4} \pm \frac{\varphi}{2}\right) \gg 1$, then the functions $F(1,\varphi)$, $F(2,\varphi)$, G $(1, \phi)$ and G $(2, \phi)$ are close to one, and Equations (23.02) change into Equations (23.01). Let us recall that the functions F and G are determined by relationships (21.22), (21.23) and (22.13).

In the direction of the principal maximum of the scattering diagram ($\phi = \alpha$), Equations (23.02) take the following form:

$$H_z = H_{0z}\left[\left(2ika\cos\alpha + \frac{1}{\cos\alpha}\right)G(1,\alpha)\,G(2,\alpha) + G(1,\alpha)\frac{\partial G(2,\alpha)}{\partial\alpha} - G(2,\alpha)\frac{\partial G(1,\alpha)}{\partial\alpha}\right]\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}},$$
$$E_z = E_{0z}\left[\left(2ika\cos\alpha - \frac{1}{\cos\alpha}\right)F(1,\alpha)F(2,\alpha) + F(1,\alpha)\frac{\partial F(2,\alpha)}{\partial\alpha} - F(2,\alpha)\frac{\partial F(1,\alpha)}{\partial\alpha}\right]\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}. \quad (23.03)$$

nce when $\alpha = \pm\frac{\pi}{2}$ we have

$$H_z = 0,$$
$$E_z = E_{0z}\left(\frac{2}{\pi}\right)^{\frac{3}{2}} e^{i\frac{3\pi}{4}}\left[2\sqrt{ka}\left(\int_0^{2\sqrt{ka}} e^{iq^2}dq + \frac{i}{4\sqrt{ka}}e^{i4ka}\right) + \frac{i}{8\sqrt{ka}}\int_0^{2\sqrt{ka}} e^{iq^2}dq\right]\frac{e^{ikr}}{\sqrt{kr}}. \quad (23.04)$$

It is interesting to observe that Expressions (23.02) to some xtent take into account, in addition to secondary diffraction, also ertiary diffraction. Actually, for the values $|\alpha| \ll \frac{\pi}{2}$ and $|\varphi| \ll \frac{\pi}{2}$, e have

$$G(1,\varphi)\,G(2,\alpha)\,e^{ika(\sin\alpha - \sin\varphi)} \approx e^{ika(\sin\alpha - \sin\varphi)} - \frac{e^{ika(2+\sin\alpha+\sin\varphi)}}{2\sqrt{\pi ka}\cos\left(\frac{\pi}{4} - \frac{\varphi}{2}\right)}e^{i\frac{\pi}{4}} - \frac{e^{ika(2-\sin\alpha-\sin\varphi)}}{2\sqrt{\pi ka}\cos\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)}e^{i\frac{\pi}{4}} + \frac{i}{4\pi ka}\,\frac{e^{ika(4-\sin\alpha+\sin\varphi)}}{\cos\left(\frac{\pi}{4} - \frac{\varphi}{2}\right)\cos\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)}. \quad (23.05)$$

he physical meaning of the four terms in the right-hand member of his equation is illustrated in Figure 48 (Figure 48a corresponds to he first term; Figure 48b corresponds to the second term, etc.).

Taking into account condition (6.15), one may write the equations or the fringing field in the left half-space ($\frac{\pi}{2} < |\varphi| < \pi$ but $|\alpha| < \frac{\pi}{2}$) n the same form as (23.02). Thus, the functions $G(1,\alpha)$, $G(2,\alpha)$ and

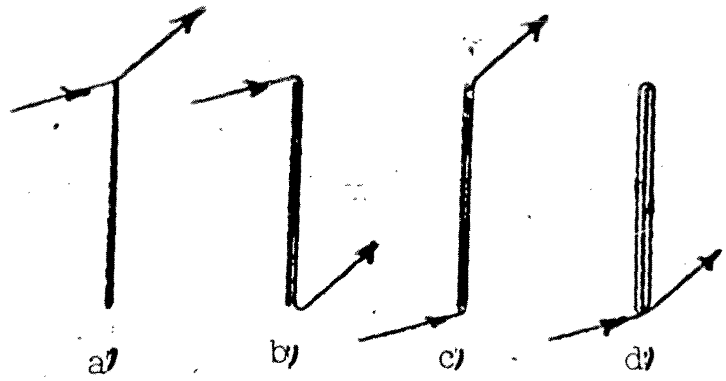

Figure 48. The schematic diagram of the waves corresponding to the various terms in Equation (22.05).

$F(1, \alpha)$, $F(2, \alpha)$ will, as before, be described by the relationships 21.22), (21.23) and (22.13). The remaining functions in Equations 22.02) will be determined when $\frac{\pi}{2} \leqslant |\varphi| \leqslant \pi$ by the following equations:

$$
\left.\begin{aligned}
G(1, \varphi) &= \frac{2}{\sqrt{\pi}} e^{-i\frac{\pi}{4}} \cdot \int\limits_{0}^{\pm 2\sqrt{ka}\sin\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)} e^{iq^2} dq, \\
G(2, \varphi) &= \frac{2}{\sqrt{\pi}} e^{-i\frac{\pi}{4}} \cdot \int\limits_{0}^{\mp 2\sqrt{ka}\sin\left(\frac{\pi}{4}-\frac{\varphi}{2}\right)} e^{iq^2} dq,
\end{aligned}\right| \qquad (23.06)
$$

$$
\left.\begin{aligned}
F(1, \varphi) &= \frac{2}{\sqrt{\pi}} e^{-i\frac{\pi}{4}} \left[\int\limits_{0}^{\pm 2\sqrt{ka}\sin\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)} e^{iq^2} dq \pm \right. \\
&\left. \pm \frac{i}{4\sqrt{ka}} \frac{e^{2ika(1+\sin\varphi)}}{\sin\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)} \right], \\
F(2, \varphi) &= \frac{2}{\sqrt{\pi}} e^{-i\frac{\pi}{4}} \left[\int\limits_{0}^{\mp 2\sqrt{ka}\sin\left(\frac{\pi}{4}-\frac{\varphi}{2}\right)} e^{iq^2} dq \mp \right. \\
&\left. \mp \frac{i}{4\sqrt{ka}} \frac{e^{2ika(1-\sin\varphi)}}{\sin\left(\frac{\pi}{4}-\frac{\varphi}{2}\right)} \right],
\end{aligned}\right| \qquad (23.07)
$$

$$g(1)=\pm\frac{\cos\frac{\alpha+\varphi}{2}+\sin\frac{\alpha-\varphi}{2}}{\sin\alpha-\sin\varphi},\quad g(2)=\pm\frac{\cos\frac{\alpha+\varphi}{2}-\sin\frac{\alpha-\varphi}{2}}{\sin\alpha-\sin\varphi},\tag{23.08}$$

$$f(1)=g(2),\quad f(2)=g(1).\tag{23.09}$$

The upper sign in Expressions (23.06) - (23.09) must be taken when $\frac{\pi}{2}\leqslant\varphi\leqslant\pi$, and the lower sign must be taken when $-\pi\leqslant\varphi\leqslant-\frac{\pi}{2}$.

Assuming $\phi = -\pi + \alpha$ (with $0\leqslant\alpha\leqslant\frac{\pi}{2}$) in Equations (23.02) and (23.06) - (23.09), let us find the field radiated by the strip in the direction toward the source

$$\left.\begin{aligned}H_z&=H_{0z}[G^2(2,\alpha)g(1)e^{i2ka\sin\alpha}+\\&+G^2(1,\alpha)g(2)e^{-i2ka\sin\alpha}]\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}},\\E_z&=E_{0z}[F^2(2,\alpha)f(1)e^{i2ka\sin\alpha}+\\&+F^2(1,\alpha)f(2)e^{-i2ka\sin\alpha}]\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}},\end{aligned}\right\}\tag{23.10}$$

where

$$g(1)=f(2)=-\frac{1+\sin\alpha}{2\sin\alpha},\quad g(2)=f(1)=\frac{1-\sin\alpha}{2\sin\alpha}.\tag{23.11}$$

When $\alpha = 0$, we have

$$\left.\begin{aligned}H_z&=-H_{0z}\left[(2ika+1)G(1,0)-2\sqrt{\frac{2ka}{\pi}}e^{i\left(2ka-\frac{\pi}{4}\right)}\right]\times\\&\times G(1,0)\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}},\\E_z&=E_{0z}\left[(2ika-1)F(1,0)+\frac{1}{\sqrt{2\pi ka}}e^{i\left(2ka+\frac{\pi}{4}\right)}\right]\times\\&\times F(1,0)\frac{e^{i\left(kr+\frac{\pi}{4}\right)}}{\sqrt{2\pi kr}}.\end{aligned}\right\}\tag{23.12}$$

Calculations of the scattering characteristics were carried out ased on the equations derived above. These scattering characteristics are the functions h(α, ϕ) and e(α, ϕ) determining the fringing ield by means of the relationships

$$\left.\begin{aligned} E_z &= E_{0z} ka e(\alpha, \varphi) \sqrt{\frac{2}{\pi kr}}\, e^{i\left(kr+\frac{3\pi}{4}\right)}, \\ H_z &= H_{0z} ka h(\alpha, \varphi) \sqrt{\frac{2}{\pi kr}}\, e^{i\left(kr+\frac{3\pi}{4}\right)}. \end{aligned}\right\} \qquad (23.13)$$

The calculations were performed for the values $ka=\sqrt{28}$ and $ka=\sqrt{80}$. In Figures 49 - 62, the following designations were used: 1) the functions h and e correspond to the rigorous theory; 2) the functions h_0 and e_0 correspond to the field from the uniform part of the current (the physical optics approach); 3) the functions h_1 and e_1 correspond to the field from the uniform and nonuniform parts of the current, but without consideration of the interaction of the edges; 4) the functions h_2 and e_2 correspond to the fringing field with consideration of secondary diffraction calculated on the basis of equations (23.13), (23.02) and (23.10). Thus, in accordance with § 6,

$$\left.\begin{aligned} e_0 &= \cos\alpha \frac{\sin[ka(\sin\alpha-\sin\varphi)]}{ka(\sin\alpha-\sin\varphi)}, \\ h_0 &= \cos\varphi \frac{\sin[ka(\sin\alpha-\sin\varphi)]}{ka(\sin\alpha-\sin\varphi)} \end{aligned}\right\} \qquad (23.14)$$

and

$$\left.\begin{matrix} e_1 \\ h_1 \end{matrix}\right\} = \frac{1}{2ka}\left\{\frac{\sin[ka(\sin\alpha-\sin\varphi)]}{\sin\frac{\alpha-\varphi}{2}} \pm i\frac{\cos[ka(\sin\alpha-\sin\varphi)]}{\cos\frac{\alpha+\varphi}{2}}\right\}, \qquad (23.15)$$

where $|\alpha|<\frac{\pi}{2}$; $|\varphi|\leq\frac{\pi}{2}$.

The results obtained show that our approximation equations agree satisfactorily with the rigorous theory already when $ka=\sqrt{28}$, although in the given case approximately one and one-half wavelengths are fitted into the width of the strip. In the direction toward the source $\left(\varphi=-\pi+\alpha,\ 0\leq\alpha\leq\frac{\pi}{2}\right)$, and also with glancing irradiation of the

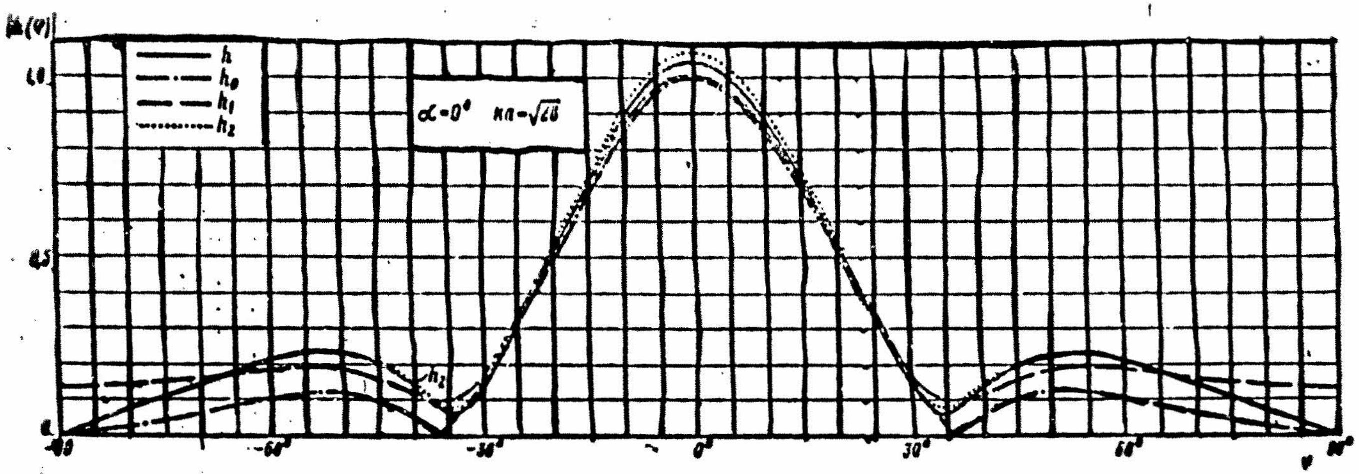

Figure 49. The scattering diagram of a field by a strip as a function of the incident angle of a plane wave (a) and the parameter $\sqrt{ka}$. The various curves correspond to various approximations.

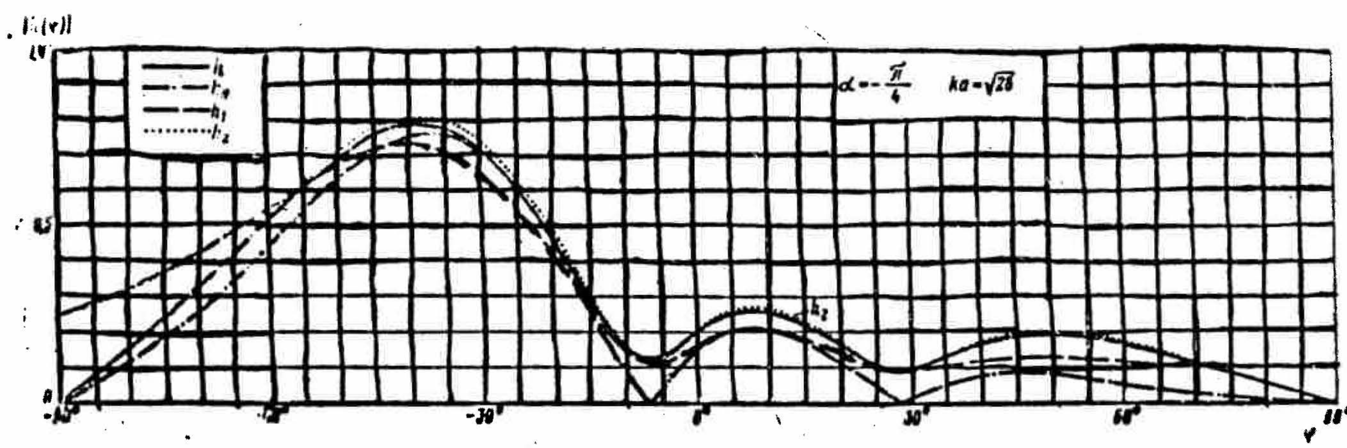

Figure 50. The same as Figure 49 with $\alpha = -\frac{\pi}{4}$.

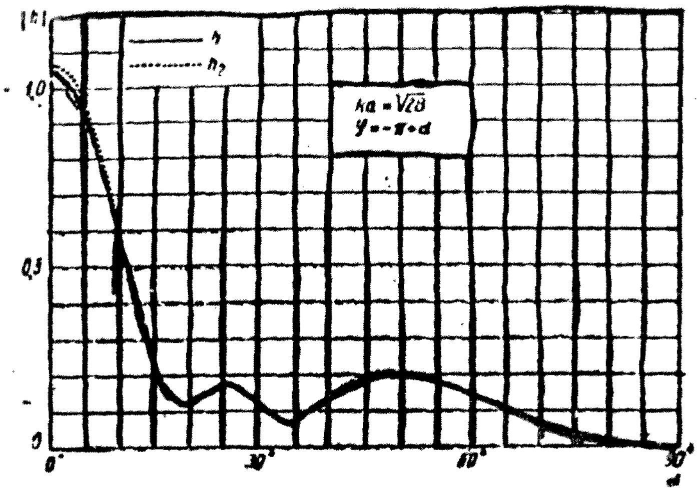

Figure 51. The same as Figure 49 when $\phi = -\pi + \alpha$.

strip ($\alpha = -\pi/2$), when the functions e_0 and h_0, and e_1 and h_1 lead to qualitatively incorrect results, the functions e_2 and h_2 give, as in the remaining cases, fully satisfactory results. Actually, the curve $|h_2|$ coincides almost everywhere with the curve $|h|$ (Figure 49-54) within the limits of graphical precision. But the calculated values of the function $|e_2|$ differ from the corresponding values of the function $|e|$ only by hundredths of a percent (Figure 55 - 62). The better agreement with the rigorous theory associated with the E-polarization is explained by the weaker interaction of the edges in this case. A certain discrepancy of the curves $|h_2|$ and $|h|$ in the vicinity of the principal scattering maximum is explained by the interpolation character of our equations.

As a consequence of the interpolation character of Equations (23.02), the integral scattering diameter obtained from Expressions (23.03) when $\alpha = 0$ does not coincide with the integral diameter found by Clemmow [46] in the form of the first terms of an asymptotic expansion in inverse powers of $\sqrt{ka}$. However, our equations, as distinct from the similar equations obtained by other authors, allow one to calculate the scattering characteristics with any incident angles of the plane wave.

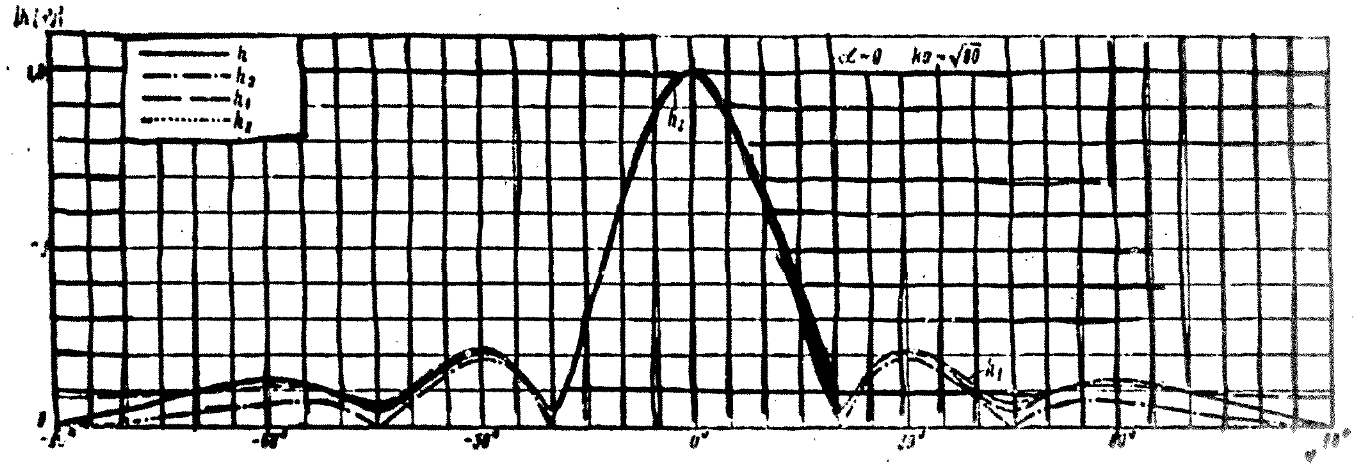

Figure 52. The function h (α, ϕ) for a strip (α = 0, ka = $\sqrt{80}$).

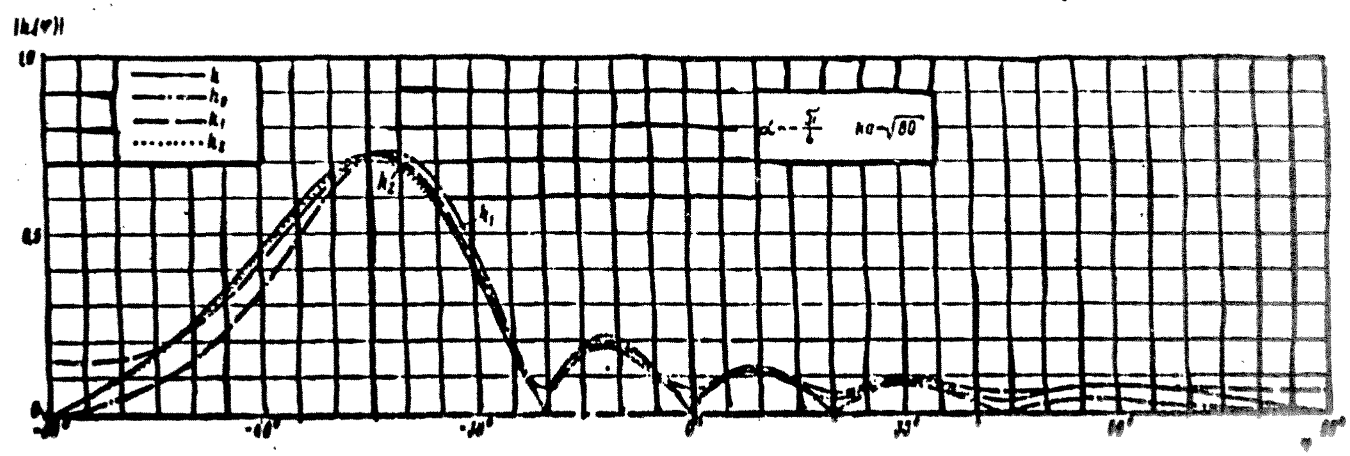

Figure 53. The same as Figure 52 when $\alpha = -\frac{\pi}{4}$.

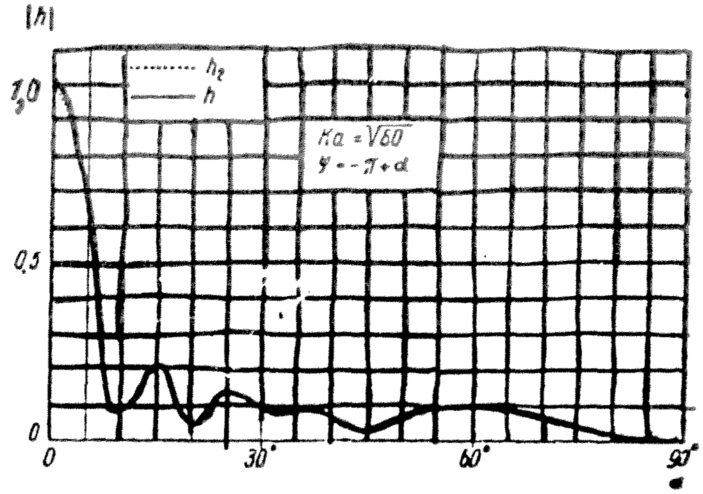

Figure 54. The same as Figure 52 when $\phi = -\pi + \alpha$.

Let us note that the functions $e(\alpha, \phi)$ and $h(\alpha, \phi)$ for Figures 49 - 52 were calculated on the basis of rigorous series which were obtained by the separation of variables in the elliptic coordinate system (compare [23])(1).

§ 24. Secondary Diffraction by a Disk

Let us refine the approximate solution of the diffraction problem for a disk which was found in Chapter II.

Let an infinitely thin, ideally conducting disk of radius a be found in free space. Let us orientate the spherical coordinate system in such a way that the normal n to the incident wave front would lie in the half-plane $\phi = \pi/2$, and form an angle $\gamma \left(0 \leq \gamma \leq \frac{\pi}{2}\right)$ with the z axis (Figure 63). Let us prescribe the incident plane wave field in

$$\mathbf{E} = \mathbf{E}_0 e^{ik(y \sin\gamma + z\cos\gamma)}, \quad \mathbf{H} = \mathbf{H}_0 e^{ik(y \sin\gamma + z\cos\gamma)}. \tag{24.01}$$

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(1) Footnote appears on page 162.

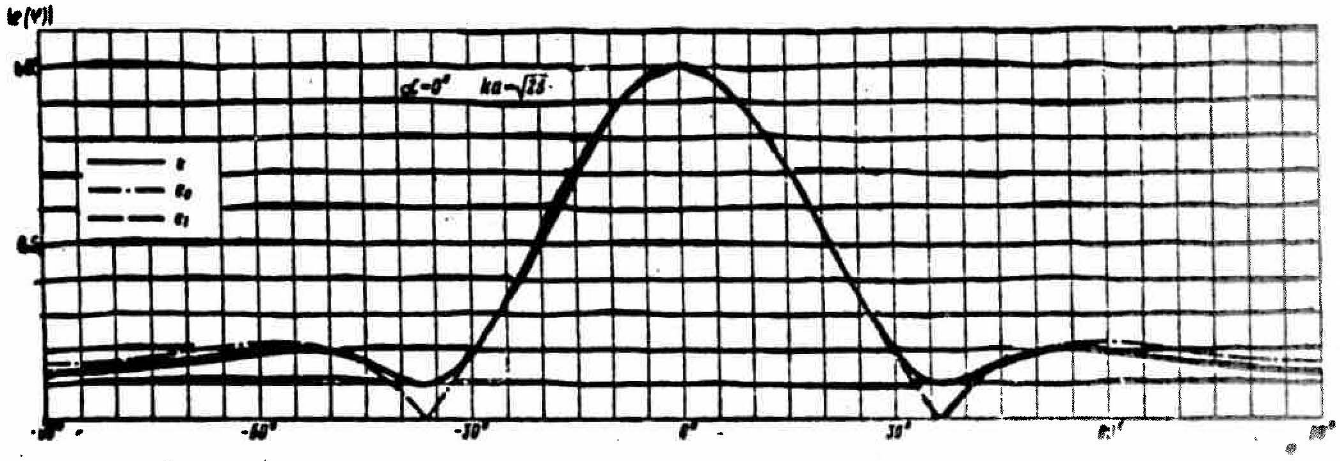

Figure 55. The function $e(\alpha, \phi)$ for a strip ($\alpha = 0$, $ka = \sqrt{28}$).

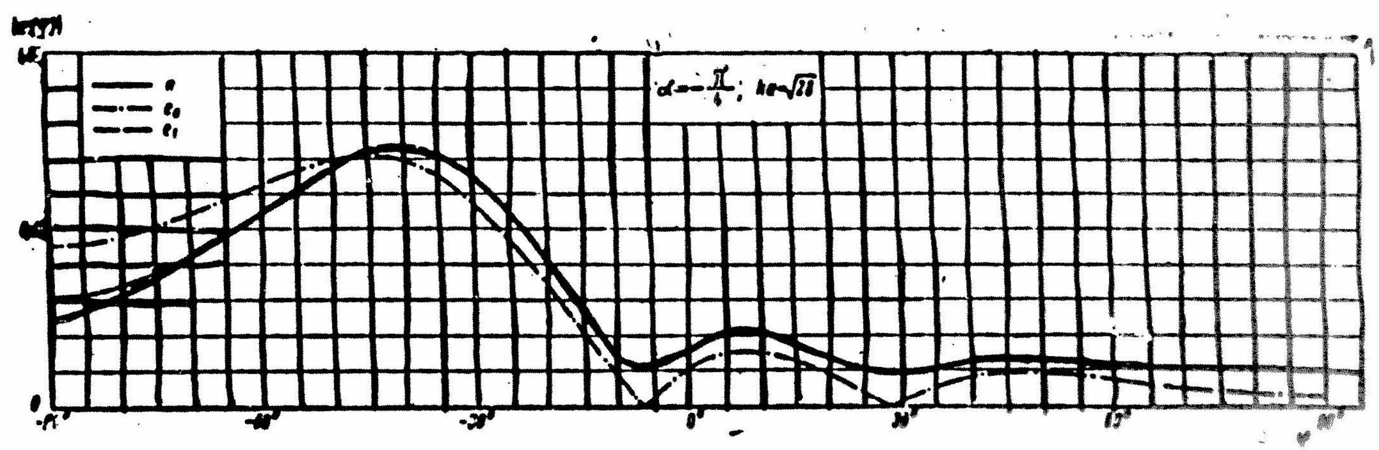

Figure 56. The same as Figure 55 when $\alpha = -\frac{\pi}{4}$.

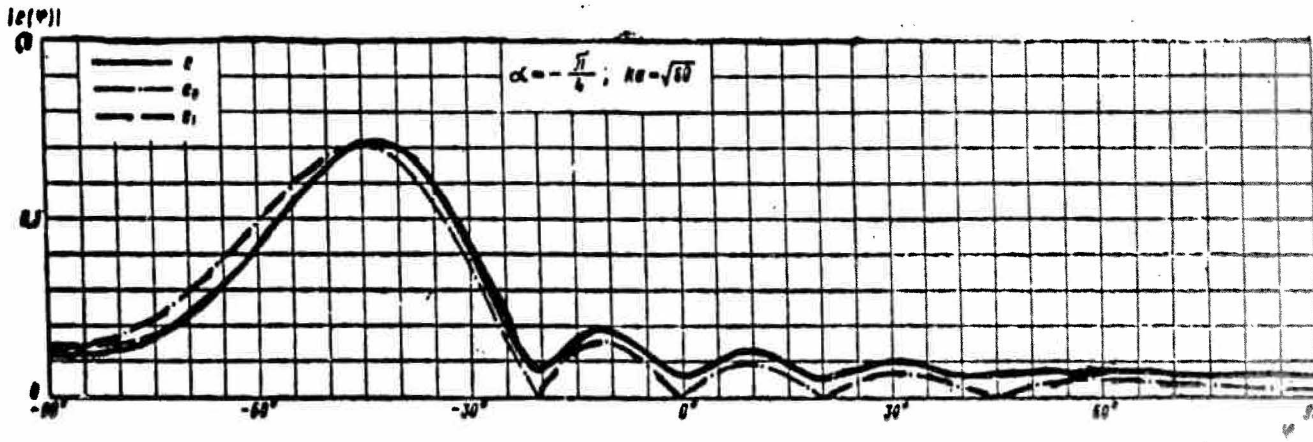

Figure 60. The same as Figure 59 when $\alpha = -\frac{\pi}{4}$.

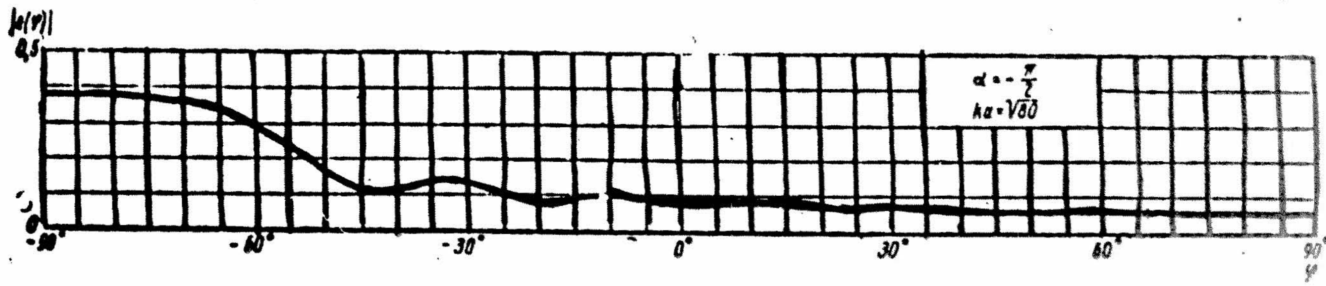

Figure 61. The same as Figure 59 when $\alpha = -\frac{\pi}{2}$.

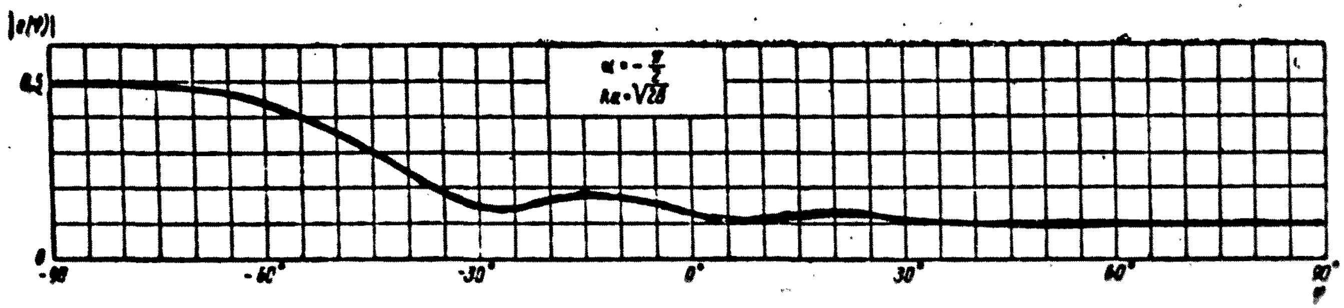

Figure 57. The same as Figure 55 when $\alpha = -\frac{\pi}{2}$.

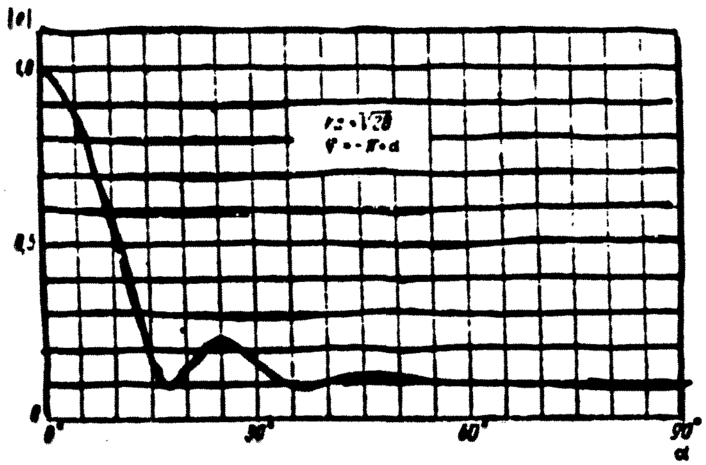

Figure 58. The same as Figure 55 when $\phi = -\pi + \alpha$.

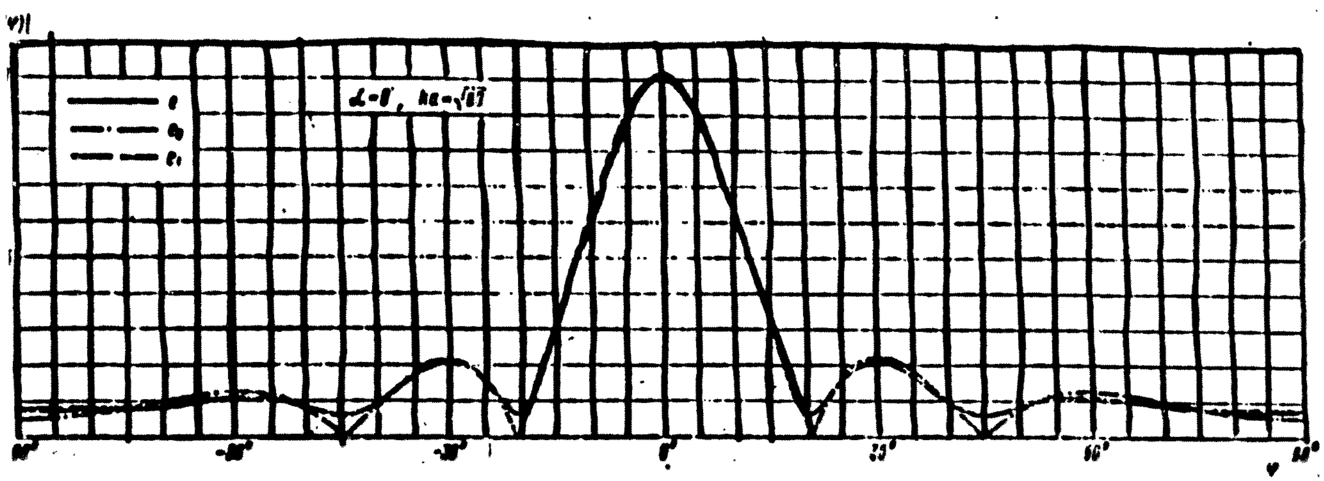

Figure 59. The function $e(\alpha, \phi)$ for a strip ($\alpha + 0$, $ka = \sqrt{80}$). The electric vector is parallel to the strip's edge.

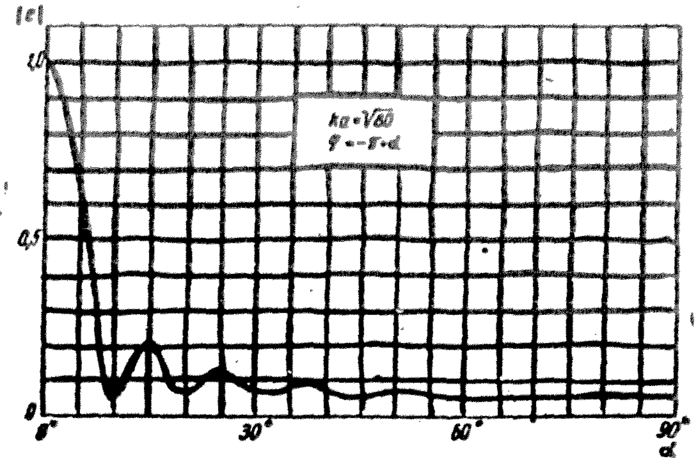

Figure 62. The same as Figure 59 when $\phi = -\pi + \alpha$.

In accordance with § 12, the fringing field in the plane $\phi = \pm\pi/2$ is described (when $R >> ka^2$) by the equations

$$\left.\begin{aligned} E_\varphi = -H_\vartheta = \frac{iaE_{0\varphi}}{2}\Big\{[f(2,\delta) - f(1,\delta)]\,J_1(\zeta) + \\ + i\,[f(2,\delta) + f(1,\delta)]\,J_2(\zeta)\Big\}\frac{e^{ikR}}{R}, \\ H_\varphi = E_\vartheta = \frac{iaH_{0\varphi}}{2}\Big\{[g(2,\delta) - g(1,\delta)]\,J_1(\zeta) + \\ + i\,[g(2,\delta) + g(1,\delta)]\,J_2(\zeta)\Big\}\frac{e^{ikR}}{R}. \end{aligned}\right\} \tag{24.02}$$

These expressions are valid when $\vartheta \leqslant \frac{\pi}{2}$ and $\gamma < \frac{\pi}{2}$. The quantities included in them are determined by the relationships:

$$\left.\begin{aligned} f(1,\delta) &= \frac{\cos\frac{\delta+\vartheta}{2} - \sin\frac{\delta-\vartheta}{2}}{\sin\delta - \sin\vartheta}, \\ f(2,\delta) &= -\frac{\cos\frac{\delta+\vartheta}{2} + \sin\frac{\delta-\vartheta}{2}}{\sin\delta - \sin\vartheta}, \\ g(1,\delta) &= -f(2,\delta), \quad g(2,\delta) = -f(1,\delta), \end{aligned}\right\} \tag{24.03}$$

$$\zeta = ka(\sin\vartheta - \sin\delta), \tag{24.04}$$

$$\sigma = \begin{cases} \Upsilon \text{ with } \varphi = \frac{\pi}{2} \\ -\Upsilon \text{ with } \varphi = -\frac{\pi}{2}. \end{cases} \qquad (24.05)$$

Let us note that here

$$\left. \begin{aligned} f(1, \delta) &= \begin{cases} f(1) \text{ with } \varphi = \frac{\pi}{2} \\ f(2) \text{ with } \varphi = -\frac{\pi}{2}, \end{cases} \\ f(2, \delta) &= \begin{cases} f(2) \text{ with } \varphi = \frac{\pi}{2} \\ f(1) \text{ with } \varphi = -\frac{\pi}{2}, \end{cases} \\ g(1, \delta) &= \begin{cases} g(1) \text{ with } \varphi = \frac{\pi}{2} \\ g(2) \text{ with } \varphi = -\frac{\pi}{2}, \end{cases} \\ g(2, \delta) &= \begin{cases} g(2) \text{ with } \varphi = \frac{\pi}{2} \\ g(1) \text{ with } \varphi = -\frac{\pi}{2}, \end{cases} \end{aligned} \right\} \qquad (24.06)$$

and the functions f(1), f(2), g(1) and g(2) are determined by the Equations (12.03) and (12.04).

When $\zeta \gg 1$, Expressions (24.02) take the form

$$\left. \begin{aligned} E_{\varphi} = -H_{\vartheta} &= \frac{iaE_{0\varphi}}{\sqrt{2\pi\zeta}} \Big[f(2, \delta)\, e^{i\left(\zeta - \frac{3\pi}{4}\right)} - \\ &\quad - f(1, \delta)\, e^{-i\left(\zeta - \frac{3\pi}{4}\right)} \Big] \frac{e^{ikR}}{R}, \\ H_{\varphi} = E_{\vartheta} &= \frac{iaH_{0\varphi}}{\sqrt{2\pi\zeta}} \Big[g(2, \delta)\, e^{i\left(\zeta - \frac{3\pi}{4}\right)} - \\ &\quad - g(1, \delta)\, e^{-i\left(\zeta - \frac{3\pi}{4}\right)} \Big] \frac{e^{ikR}}{R}. \end{aligned} \right\} \qquad (24.07)$$

They show that the fringing field in this region may be investigated as the sum of spherical waves from two luminous points on the rim of the disk with the polar angle $\psi = \pm\pi/2$. The diffraction by a disk of each of these waves may be studied as was done in the case of a strip but we shall proceed differently.

Starting from Expressions (24.18) and (24.19), it is not difficult to write interpolation equations for the fringing field which are suitable for any values of γ and ϑ in the interval (0; π/2), but when ϕ = ±/2;

$$E_\varphi = -H_\vartheta = \frac{iaE_{0\varphi}}{2}\Big\{[F(2,\vartheta)F(1,\delta)f(2,\delta) - F(1,\vartheta)F(2,\delta)f(1,\delta)]J_1(\zeta) + i[F(2,\vartheta)F(1,\delta)f(2,\delta) + F(1,\vartheta)F(2,\delta)f(1,\delta)]J_2(\zeta)\Big\}\frac{e^{ikR}}{R}, \qquad (24.20)$$

$$E_\vartheta = H_\varphi = \frac{iaH_{0\varphi}}{2}\Big\{[G(2,\vartheta)G(1,\delta)g(2,\delta) - G(1,\vartheta)G(2,\delta)g(1,\delta)]J_1(\zeta) + i[G(2,\vartheta)G(1,\delta)g(2,\delta) + G(1,\vartheta)G(2,\delta)g(1,\delta)]J_2(\zeta)\Big\}\frac{e^{ikR}}{R}. \qquad (24.21)$$

Let us note that when $\gamma = 0$ these expressions will be valid for any values of the azimuth ϕ, since then any point of space may be considered to be located in the incident plane.

In the direction of the scattering diagram's principal maximum — that is, when $\vartheta = \gamma,\ \varphi = \frac{\pi}{2}$ — the fringing field (24.20) and (24.21) takes the form

$$\left.\begin{aligned} E_\varphi &= -H_\vartheta = \frac{ika^2}{2}E_{0\varphi}\cdot F(2,\gamma)F(1,\gamma)\frac{e^{ikR}}{R}\cos\gamma, \\ E_\vartheta &= H_\varphi = \frac{ika^2}{2}H_{0\varphi}\cdot G(2,\gamma)G(1,\gamma)\frac{e^{ikR}}{R}\cos\gamma. \end{aligned}\right\} \qquad (24.22)$$

However, these expressions have an interpolation character, and with small values of the angle γ it is impossible to consider them to be more precise than the simple equations of § 9 and § 12. In particular, with $\gamma = 0$, when the fringing field must not depend on the incident wave polarization, they give values which are different for the E-polarization and H-polarization by small quantities of the order of $\frac{1}{\sqrt{ka}}$. Therefore, in this case (when $\gamma = 0$) it makes sense to use Expressions (24.20) and (24.21) only far from the z axis, switching to Equations (24.02) near the z axis.

In the case when the current filament passes through the point Q parallel to the half-plane's edge, the field at the point P is determined — in accordance with (20.11) and (20.12) — by the equations

$$\left.\begin{aligned} E_z &= ik^2 p_z\left[u(d,\varphi'-\varphi'')-u(d,\varphi'+\varphi'')\right]\sqrt{\frac{2\pi}{kR}}\,e^{i\left(kR-\frac{\pi}{4}\right)},\\ H_z &= ik^2 m_z\left[u(d,\varphi'-\varphi'')+u(d,\varphi'+\varphi'')\right]\sqrt{\frac{2\pi}{kR}}\,e^{i\left(kR-\frac{\pi}{4}\right)}. \end{aligned}\right\} \qquad (24.15)$$

With the absence of a half-plane, these sources create at the point P the field

$$\left.\begin{aligned} E_z &= ik^2 p_z\sqrt{\frac{2\pi}{kR}}\,e^{i\left(kR-\frac{\pi}{4}\right)}\,e^{-ikd\cos(\varphi'-\varphi'')},\\ H_z &= ik^2 m_z\sqrt{\frac{2\pi}{kR}}\,e^{i\left(kR-\frac{\pi}{4}\right)}\,e^{-ikd\cos(\varphi'-\varphi'')}. \end{aligned}\right\} \qquad (24.16)$$

Comparing Equations (24.15) and (24.16), we obtain the same Expression (24.14) for the shading functions. Consequently, a spherical wave in the direction perpendicular to an ideally conducting half-plane is shaded by it the same as a cylindrical wave.

Let us note, however, that Expressions (24.14) are not equivalent to Expressions (21.22), (21.23) and (22.13), since the first represent the shading function by a half-plane of a wave from a single source, and the latter represent the shading function of an edge wave which we approximate by waves from two sources located on both sides of the corresponding half-plane. Since the shading functions of spherical and cylindrical waves are the same, the edge wave shading functions of a strip and a disk also will coincide.

Therefore, the approximation expressions for a field scattered by a disk which take account of secondary diffraction may be represented in the region $\varphi=\pm\frac{\pi}{2}$, $0<\vartheta<\frac{\pi}{2}$ (with $\zeta \gg 1$) in the following form:

$$E_\varphi = -H_\vartheta = \frac{iaE_{0\varphi}}{\sqrt{2\pi\zeta}}\left[F(2,\vartheta)f(2,\delta)e^{i\left(\zeta-\frac{3\pi}{4}\right)} - F(1,\vartheta)f(1,\delta)e^{-i\left(\zeta-\frac{3\pi}{4}\right)}\right]\frac{e^{ikR}}{R},$$
$$H_\varphi = E_\vartheta = \frac{iaH_{0\varphi}}{\sqrt{2\pi\zeta}}\left[G(2,\vartheta)g(2,\delta)e^{i\left(\zeta-\frac{3\pi}{4}\right)} - G(1,\vartheta)g(1,\delta)e^{-i\left(\zeta-\frac{3\pi}{4}\right)}\right]\frac{e^{ikR}}{R}, \quad (24.17)$$

where the functions F and G are obtained from Equations (21.22), (21.23) and (22.13) by the replacement of ϕ by ϑ. Equations (24.17) may be investigated as the asymptotic representation (with $\zeta \gg 1$) of the following expressions:

$$E_\varphi = -H_\vartheta = \frac{iaE_{0\varphi}}{2}\left\{[F(2,\vartheta)f(2,\delta) - F(1,\vartheta)f(1,\delta)]J_1(\zeta) + i[F(2,\vartheta)f(2,\delta) + F(1,\vartheta)f(1,\delta)]J_2(\zeta)\right\}\frac{e^{ikR}}{R},$$

$$H_\varphi = E_\vartheta = \frac{iaH_{0\varphi}}{2}\left\{[G(2,\vartheta)g(2,\delta) - G(1,\vartheta)g(1,\delta)]J_1(\zeta) + i[G(2,\vartheta)g(2,\delta) + G(1,\vartheta)g(1,\delta)]J_2(\zeta)\right\}\frac{e^{ikR}}{R}. \quad (24.18)$$

These expressions hold in the region $\vartheta \leqslant \frac{\pi}{2}$ for the values $\vartheta \approx \frac{\pi}{2}$ and $\gamma \ll \frac{\pi}{2}$. Using Equation (24.18), let us write the expressions for the field radiated by a disk in the direction γ when a plane wave is incident on it (from left to right) at an angle ϑ

$$E_\varphi = -H_\vartheta = \frac{iaE_{0\varphi}}{2}\left\{-[F(2,\delta)f(1,\delta) - F(1,\delta)f(2,\delta)]J_1(\zeta) + i[F(2,\delta)f(1,\delta) + F(1,\delta)f(2,\delta)]J_2(\zeta)\right\}\frac{e^{ikR}}{R},$$
$$H_\varphi = E_\vartheta = \frac{iaH_{0\varphi}}{2}\left\{-[G(2,\delta)g(1,\delta) - G(1,\delta)g(2,\delta)]J_1(\zeta) + i[G(2,\delta)g(1,\delta) + G(1,\delta)g(2,\delta)]J_2(\zeta)\right\}\frac{e^{ikR}}{R}. \quad (24.19)$$

Here $\gamma \approx \frac{\pi}{2}\left(\gamma \leqslant \frac{\pi}{2}\right)$ and $\vartheta \ll \frac{\pi}{2}$.

Equations (24.20) and (24.21) have the following important properties, They do not have discontinuities, they include the case of glancing incidence of a plane wave, and they satisfy the reciprocity principle. From them it follows that $E_\varphi = H_\varphi = 0$ when $\vartheta = \frac{\pi}{2}$ — that is, the fringing field does not experience a discontinuity on the plane z = 0. Moreover, $E_\vartheta = H_\varphi = 0$ with any values of ϑ, if $\gamma = \frac{\pi}{2}$, — that is, a plane wave polarized perpendicular to the disk's plane does not experience diffraction with glancing irradiation of the disk.

As in the case of diffraction by a strip, the new approximation expressions consider to some extent tertiary diffraction [see Equations (23.05) and Figure 48].

Using Condition (9.04), it is not difficult to write equations for the fringing field in the left half-space $\left(\frac{\pi}{2} < \vartheta < \pi;\ \varphi = \pm\frac{\pi}{2}\right)$

$$E_\varphi = -H_\vartheta = \frac{iaE_{0\varphi}}{2}\Big\{[F(2, \pi-\vartheta)F(1,\delta)f(2,\delta) - \\ - F(1, \pi-\vartheta)F(2,\delta)f(1,\delta)]J_1(\zeta) + \\ + i[F(2,\pi-\vartheta)F(1,\delta)f(2,\delta) + \\ + F(1,\pi-\vartheta)F(2,\delta)f(1,\delta)]J_2(\zeta)\Big\}\frac{e^{ikR}}{R}, \tag{24.23}$$

$$E_\vartheta = H_\varphi = \frac{iaH_{0\varphi}}{2}\Big\{[G(2,\pi-\vartheta)G(1,\delta)g(2,\delta) - \\ - G(1,\pi-\vartheta)G(2,\delta)g(1,\delta)]J_1(\zeta) + \\ + i[G(2,\pi-\vartheta)G(1,\delta)g(2,\delta) + \\ + G(1,\pi-\vartheta)G(2,\delta)g(1,\delta)]J_2(\zeta)\Big\}\frac{e^{ikR}}{R}, \tag{24.23}$$

where the functions f and g are determined by the equations

$$\left.\begin{aligned} g(1,\delta) = f(2,\delta) = \frac{\cos\frac{\delta+\vartheta}{2} + \sin\frac{\delta-\vartheta}{2}}{\sin\delta - \sin\vartheta}, \\ f(1,\delta) = g(2,\delta) = \frac{\cos\frac{\delta+\vartheta}{2} - \sin\frac{\delta-\vartheta}{2}}{\sin\delta - \sin\vartheta}. \end{aligned}\right\} \tag{24.24}$$

In the direction towards the source $\left(\vartheta = \pi - \gamma,\ \varphi = -\frac{\pi}{2}\right)$, the fringing field equals

$$
\left.
\begin{aligned}
E_\varphi = -H_\vartheta = \frac{iaE_{0x}}{2}\Big\{&[F^2(1,\delta)f(2,\delta) - \\
- F^2(2,\delta)f(1,\delta)]J_1(\zeta) + i[&F^2(1,\delta)f(2,\delta) + \\
+ F^2(2,\delta)f(1,\delta)]J_2(\zeta)\Big\}&\frac{e^{ikR}}{R}, \\
E_\vartheta = H_\varphi = \frac{iaH_{0x}}{2}\Big\{&[G^2(1,\delta)g(2,\delta) - \\
- G^2(2,\delta)g(1,\delta)]J_1(\zeta) + i[&G^2(1,\delta)g(2,\delta) + \\
+ G^2(2,\delta)g(1,\delta)]J_2(\zeta)\Big\}&\frac{e^{ikR}}{R},
\end{aligned}
\right\} \tag{24.25}
$$

where

$$
\left.
\begin{aligned}
\delta &= -\gamma; \\
f(1,\delta) = g(2,\delta) &= -\frac{1+\sin\gamma}{2\sin\gamma}; \\
f(2,\delta) = g(1,\delta) &= \frac{1-\sin\gamma}{2\sin\gamma}.
\end{aligned}
\right\} \tag{24.26}
$$

As was already noted, it makes sense to use Equations (24.25) only far from the z axis, changing to Expression (12.15) of the previous approximation in the vicinity of the z axis. A calculation of functions $\Sigma(\pi-\gamma)$ and $\bar{\Sigma}(\pi-\gamma)$ (Figures 65 and 66) which determine the effective scattering surface [see Expressions (12.17)] was performed on the basis of these equations when ka = 5. A comparison was carried out of this calculation with the results of measurements. The two experimental diagrams (the dashed lines)[(2)] depicted in Figure 65 characterize the experimental precision. As distinct from the previous approximations, which lead in this case to qualitatively incorrect results [see Equations (10.06), (10.07) and (12.15)], we observe a satisfactory agreement of theory with experiment.

For verifying the results obtained, a calculation was also carried out of the functions $V^{(1)}(\vartheta)$ and $V^{(2)}(\vartheta)$ [see Equations (9.07)] when ka = 5 (Figure 67 and 68) with normal irradiation of a disk by a plane wave. Curve 1 corresponds to the field calculated from the rigorous theory [34]; curve 2 corresponds to the field from

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(2) Footnote appears on page 162.

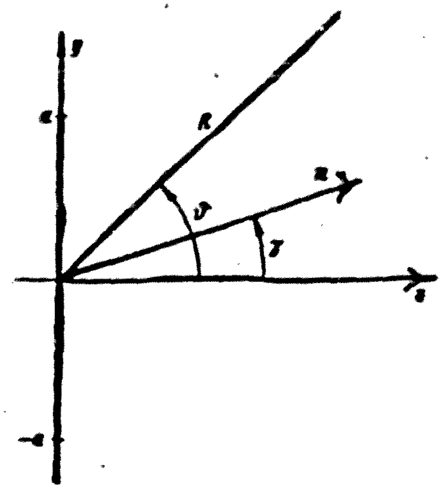

Figure 63. The cross section of a disk with the plane yOz; n is the normal to the incident wave front.

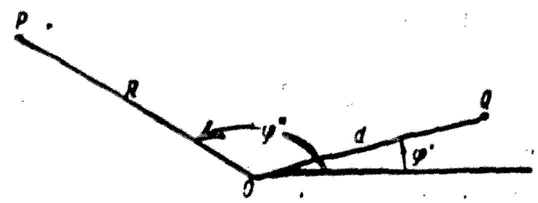

Figure 64. Excitation of a half-plane by an elementary dipole which is located at the point Q.

Let us compare the shading of spherical and cylindrical waves by a half-plane. Let an ideally conducting half-plane be found in free space, and let there be an elementary dipole at the point Q (Figure 64). Let us find the field in the plane perpendicular to the half-plane's edge and passing through the point Q.

In accordance with the reciprocity principle, it is determined for the electric dipole by the relationship

$$E_z = \frac{p_z}{p_{0z}} E_z(Q), \tag{24.08}$$

and for the magnetic dipole by the relationship

$$H_z = \frac{m_z}{m_{0z}} H_z(Q). \tag{24.09}$$

Here p_z (m_z) is the electric (magnetic) dipole moment found at the point Q; p_{oz} and m_{oz} are the moments of the auxiliary dipoles which are placed at the point P; $E_z(Q)$ and $H_z(Q)$ are the fields created by the auxiliary dipoles at the point Q.

Now let us remove the auxiliary dipoles to such a distance that the spherical wave arriving from them may be considered to be a plane wave on the section from the half-plane's edge to the point Q. In this case, in accordance with Equation (20.08), the field created by the wave at the point Q will equal

$$\left.\begin{aligned} E_z(Q) &= E_{0z}(0)\left[u(d, \varphi'-\varphi'') - u(d, \varphi'+\varphi'')\right], \\ H_z(Q) &= H_{0z}(0)\left[u(d, \varphi'-\varphi'') + u(d, \varphi'+\varphi'')\right]. \end{aligned}\right\} \tag{24.10}$$

The expressions

$$E_{0z}(0) = k^2 p_{0z}\frac{e^{ikR}}{R}, \quad H_{0z}(0) = k^2 m_{0z}\frac{e^{ikR}}{R} \tag{24.11}$$

determine the fields created by the auxiliary dipoles in free space (with the absence of the half-plane) at the point 0.

Consequently, the fields excited at the point P by the electric and magnetic dipoles which are found at the point Q above the half-plane equal respectively

$$\left.\begin{aligned} E_z &= k^2 p_z\left[u(d, \varphi'-\varphi'') - u(d, \varphi'+\varphi'')\right]\frac{e^{ikR}}{R}, \\ H_z &= k^2 m_z\left[u(d, \varphi'-\varphi'') + u(d, \varphi'+\varphi'')\right]\frac{e^{ikR}}{R}. \end{aligned}\right\} \tag{24.12}$$

With the absence of the half-plane, these dipoles create at the point P the field

$$\left.\begin{aligned} E_z &= k^2 p_z\frac{e^{ikR}}{R}e^{-ikd\cos(\varphi'-\varphi'')}, \\ H_z &= k^2 m_z\frac{e^{ikR}}{R}e^{-ikd\cos(\varphi'-\varphi'')}. \end{aligned}\right\} \tag{24.13}$$

Comparing Expressions (24.12) and (24.13) we find the shading functions

$$\left.\begin{aligned} \bar{F} &= \left[u(d, \varphi'-\varphi'') - u(d, \varphi'+\varphi'')\right]e^{ikd\cos(\varphi'-\varphi'')}, \\ \dot{G} &= \left[u(d, \varphi'-\varphi'') + u(d, \varphi'+\varphi'')\right]e^{ikd\cos(\varphi'-\varphi'')}. \end{aligned}\right\} \tag{24.14}$$

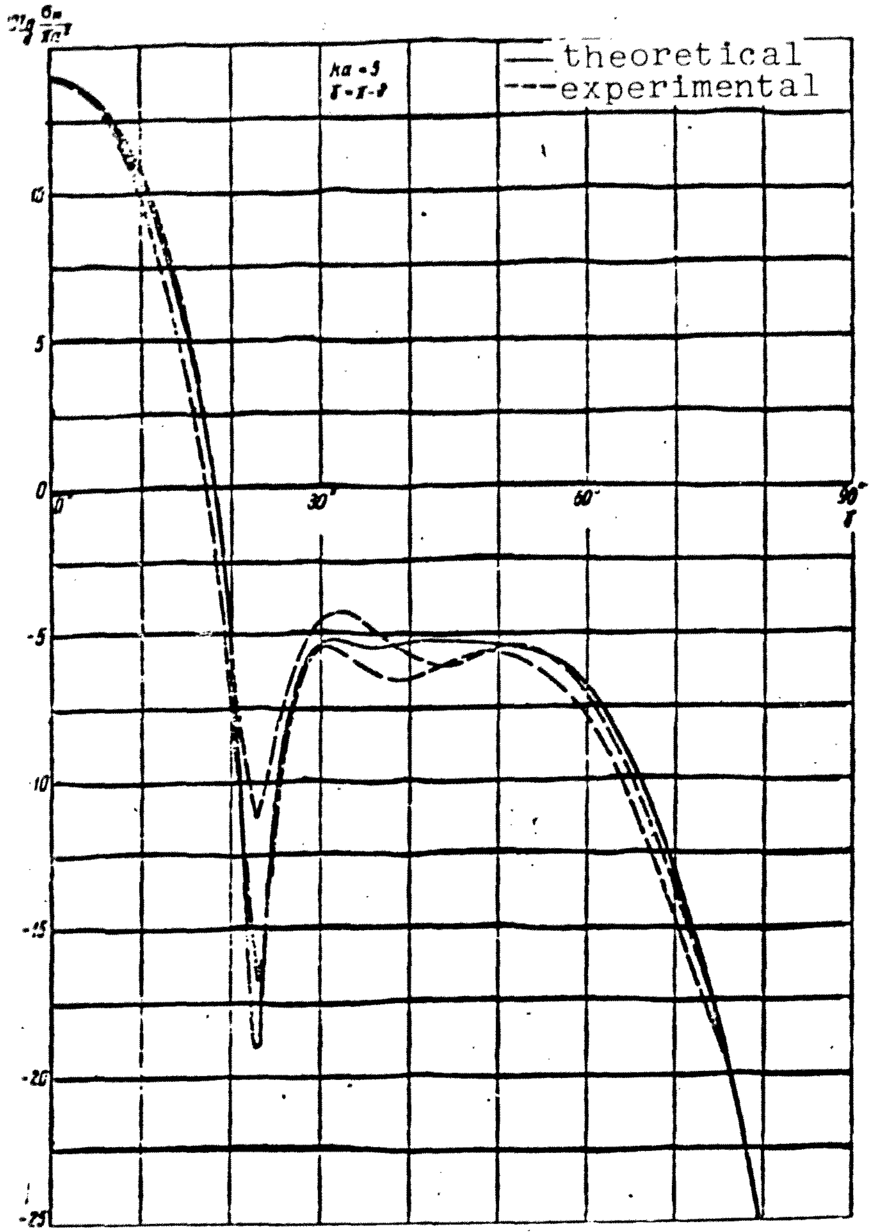

Figure 65. The diagram of a disk's effective scattering surface when the plane wave's magnetic vector is perpendicular to the incident plane.

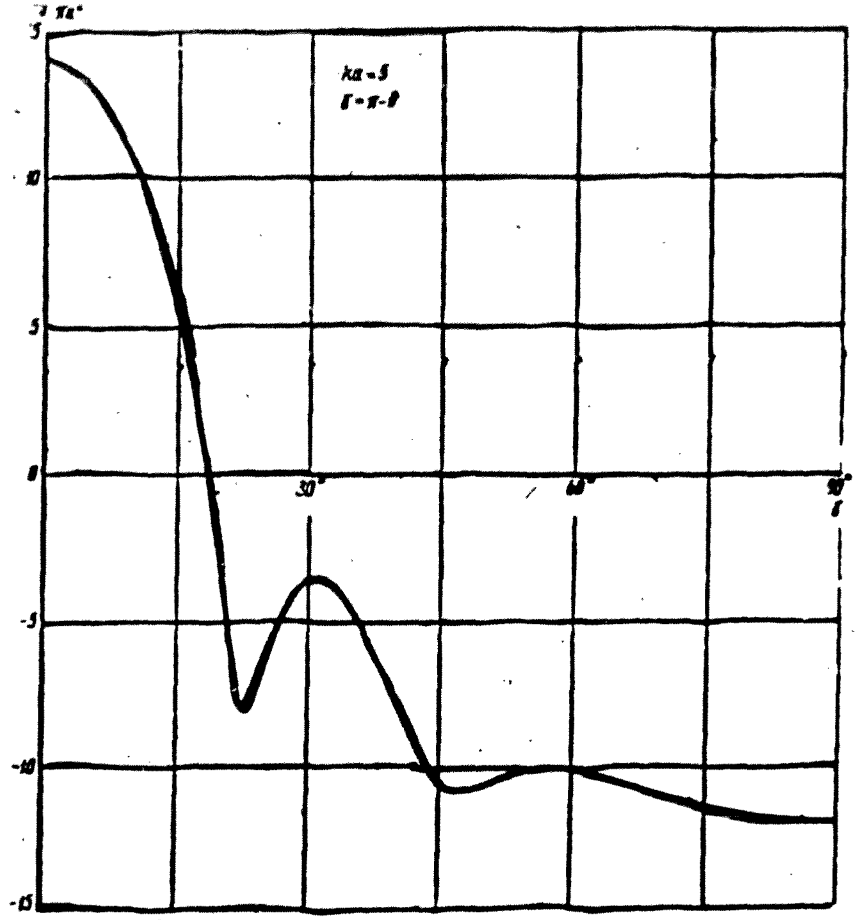

Figure 66. The calculated diagram of a disk's effective scattering surface when the plane wave's electric vector is perpendicular to the incident plane.

the uniform part of the current (the physical optics approach). Curve 3 corresponds to the field from the uniform and nonuniform parts of the current, but without the interaction of the edges. Curve 4 corresponds to the field with consideration of secondary diffraction. As is seen from these graphs, consideration of the edge interaction refines the previous approximation and ensures better agreement with the rigorous theory results.

The problem of secondary diffraction by a cylinder may be solved by a similar method. However, considering that the corrections which

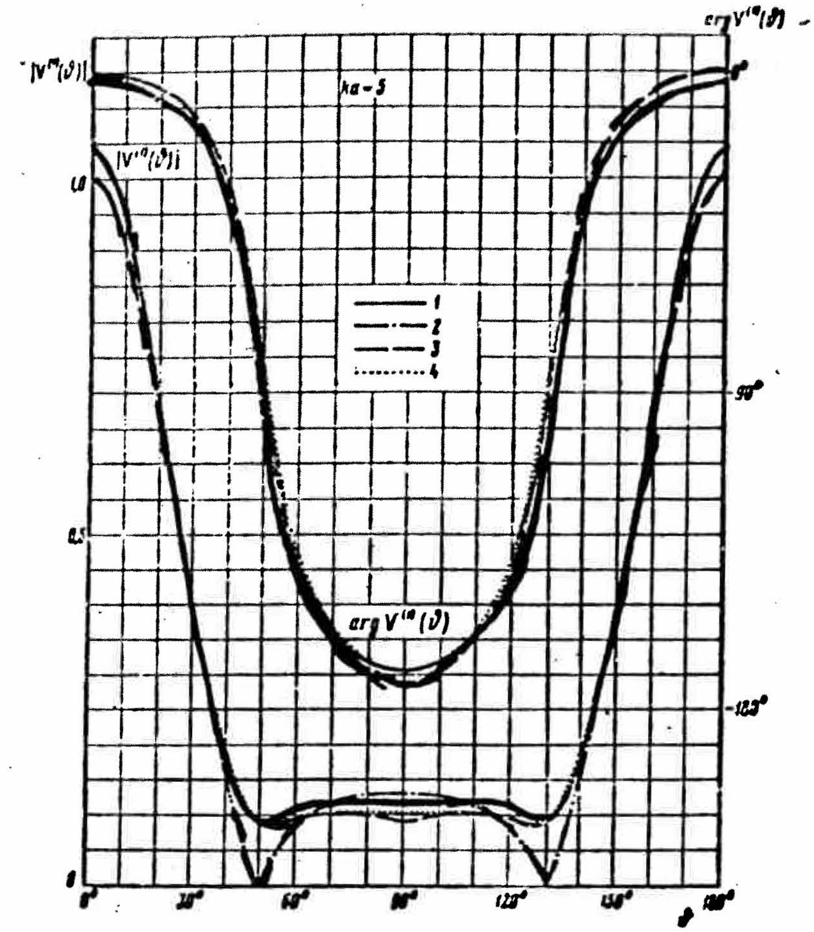

Figure 67. The function $V^{(1)}(\vartheta)$ for a disk with normal incidence of a plane wave (curve 4). Curves 1, 2 and 3 from Figure 20 are drawn for comparison.

depend on the secondary diffraction here are small (on the order of 1 dB) when $ka = \pi$, $kl = 10\pi$, and the equations are substantially more complicated, we shall not cite them here.

In the problems investigated above, the edge waves have the character of cylindrical or spherical waves — that is, they decrease rather rapidly with the distance from the edge. Therefore, in the case when the linear dimensions of the faces are approximately two wavelengths, it is sufficient to limit ourselves to a consideration

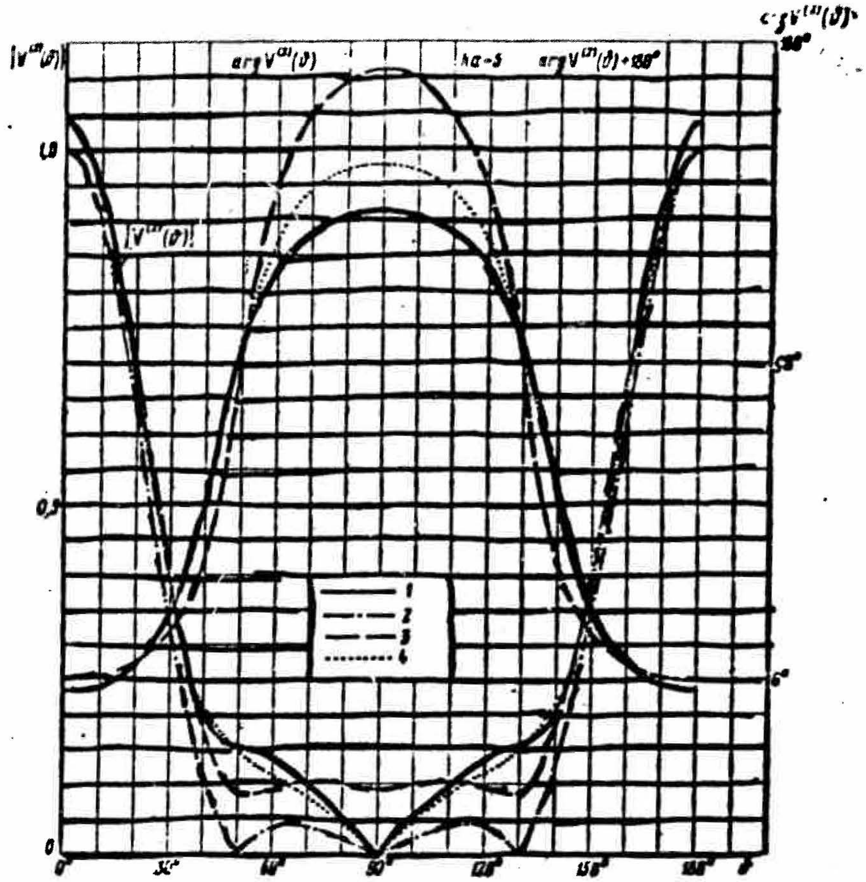

Figure 68. The function $V^{(2)}(\vartheta)$ for a disk with the normal incidence of a plane wave (curve 4). Curves 1, 2 and 3 from Figure 21 are drawn for comparison

of only secondary waves. In Chapter VII we will investigate the problem of a dipole in which the edge waves decrease so slowly that it is necessary to consider multiple diffraction.

§ 25. A Brief Review of the Literature

In this and previous chapters, approximation expressions were obtained for the scattering characteristics of a plane wave by various bodies. These expressions were derived with the help of physical considerations which do not pretend to be mathematically rigorous, and they are adequate for sufficiently short waves. In the literature there are a number of works in which similar results were obtained. A majority of these works also are not characterized by mathematical rigor, and they are based on certain physical assumptions. Therefore, one may relate them to the physical theory of diffraction. Only in a few works (related to the simpler diffraction problems) did they succeed in obtaining specific results at a higher level of mathematical rigor — more precisely, while developing asymptotic methods of mathematical diffraction theory.

We will briefly list the most important results obtained in a number of papers and books, grouping the material in the following sequence:

1. Diffraction by plane, infinitely thin plates (an infinite strip, a circular disk) and diffraction by auxiliary apertures in a flat screen (an infinite slit, a circular hole).

2. Diffraction by three-dimensional bodies with edges (a finite cylinder, a finite cone, etc.).

3. Other diffraction problems.

When investigating the first group of diffraction problems, it is necessary to keep in mind the principle of duality [4] which enable one to easily change from a strip to a slit, from a disk to a circular hole, etc. In the literature as a rule, they preferred to investigate apertures in an infinite flat screen, whereas in our book, diffraction by a strip and a disk was studied. This approach facilitates the transition to three-dimensional bodies (see the remarks at the beginning of this chapter).

Based on the time of appearance (if we do not consider the works of Schwarzschild [15] which we talked about in the Introduction), one should first of all mention the works of Braunbek [28 - 30] which were devoted to the diffraction of a scalar wave by a circular hole in a flat screen. Assuming that the plane wave is incident normal to the screen, the author obtained an approximation solution in the form of a surface integral. The boundary values of the integrand were taken from the rigorous solution to the problem of diffraction by a half-plane which was found by Sommerfeld. The field was calculated in the far zone on the axis of the hole and far from it, and also on the axis near the screen. Using this approach, Braunbek recently solved the problem of scalar wave diffraction by an aperture in a concially shaped screen [31].

In the papers of Frahn [32, 33], this method was used for the diffraction of electromagnetic waves. Diffraction of a plane wave incident normal to an ideally conducting screen with a circular hole was investigated. The field was calculated in the hole and on the axis, and also the field in the far zone and the transmission coefficient (the ratio of the energy passing through the hole to the energy falling on it) were calculated.

In these works of Braunbek and Frahn, secondary diffraction was not considered. The expressions obtained by them for the fringing field intensity in the far zone agree with similar expressions following from our equations (§ 9).

Karp and Russek [51] studied diffraction by a slit in the case when the incident wave's electric vector is parallel to the slit edge. They investigated each semi-infinite part of the screen as a half-plane excited by the incident wave field and a "virtual" source localized on the edge of the opposite half-plane. The moments of these sources were determined from a system of two algebraic equations which were obtained by using the asymptotic expressions resulting from the rigorous solution for the half-plane. Secondary diffraction was considered, and partially the general interaction. Special attention was allotted to calculating the transmission coefficient, but equations for the scattering characteristics which would be suitable with all directions of incident wave propagation were absent.

Clemmow [46] and Millar [47 - 49] in their works calculated the transmission coefficients with normal irradiation of a slit and a hole, and also the field in the hole. The solution was sought by means of curvilinear integrals of the fictitious linear currents on the aperture edges. The interaction of the edges was considered. The case of inclined irradiation was not investigated, since it turned out to be too complicated for investigation by this method.

The "geometric theory of diffraction" of Keller [42 - 44] which deals with diffraction rays is of special interest. The phase and amplitutude corresponding to each diffraction ray are determined at each ray point on the basis of geometric considerations and the law of the conservation of energy. The initial diffraction ray amplitude is assumed to be proportional to the incident ray amplitude at the point of its diffraction. The unknown proportionality constant between the amplitudes and the initial phase difference is determined from a comparison with the results of well-known solutions of diffraction problems. In this way, the fields scattered with the normal incidence of a plane wave on a slit and hole in a flat screen are found. These fields are obtained with consideration of multiple diffractions, but they are not precise wave equation solutions, since their calculation was started from approximation relations. Moreover, geometric diffraction theory is not applicable near caustics, and also in the vicinity of the scattering diagram principal maximum.

In a recently published paper of Buchal and Keller [52], a new method for the solution of diffraction problems for holes in a flat screen was proposed. The caustics and shadow boundaries here are investigated as thin boundary layers, inside of which a rapid field change takes place. This method supplements geometric diffraction theory, and in particular enables one to find the field at caustics and on the shadow boundary.

Recently, the method of integral equations has been applied to the solution of diffraction problems of holes in a flat screen. In particular, Greenberg [53, 54] reduced the solution of this problem to an integral equation for a "shadow" current which is, in our terminology, half the nonuniform part of the current. The resulting integral equations may be solved (with any ratio between the dimensions of the hole and the wavelength) by the method of successive approximations. Moreover, they allow one to obtain asymptotic expressions which are suitable for short waves. In Reference [55] Greenberg found an asymptotic expression for the current on a strip with $ka \gg 1$ ($2a$ is the strip's width). Greenberg and Pimenov [56] obtained a similar solution in the case of normal incidence of a plane wave on a circular hole. Using the same method, an asymptotic expression was found for the current on a flat ring [57], the width and inner diameter of which are a great deal larger than the wavelength.

The above listed works [53 - 57] already relate to the mathematical theory of diffraction: in them the first terms of the asymptotic expansions for the current were obtained with the desire evidently to also be able to calculate the following terms. Unfortunately, the asymptotic expressions which have been found up to now refer only to currents, and one is obliged to calculate the scattering characteristics by means of numerical quadratures [56]. As a consequence of the rapid oscillation of the integrands, such a method leads to rather unwieldy calculations and does not enable one to formulate a clear representation of the fringing field formation, and also does not allow one to study this field properly.

Millar [58] investigated the problems of electromagnetic wave diffraction by slits in a flat screen. The system of integral equations obtained by him for the current is solved by the method of successive approximations. The field in the hole is calculated ~~from~~ the currents which are found, and then on the basis of the field in the hole the field in the far zone and the transmission coefficient are calculated. All the indicated quantities are represented in the form of an asymptotic expansion in reciprocal powers of the parameter $\sqrt{ka}$. A solution also is obtained in the case of glancing incidence of a plane wave.

Let us note that the asymptotic expressions obtained by the method of integral equations are distinguished by their considerable complexity, and frequently require tabulation of the new special functions appearing in the expressions.

In the recently issued volume of Handbuch der Physik [50], which is devoted to diffraction theory, the complex characteristic of plane wave scattering by a strip was studied directly, omitting the calculation of the currents. For this characteristic, a singular integral equation was formulated, the solution of which was sought in the form of an asymptotic series in reciprocal powers of $\sqrt{ka}$. The first term of the series corresponds to Equations (6.14) and (6.16). The following term takes into account the interaction of the edges, and becomes infinite with the glancing incidence of a plane wave and also for observation points lying in the strip's plane. Therefore, the simple expressions obtained in [50] do not allow one to construct the complete scattering characteristic. In [50] diffraction by a disk, a sphere, and an infinite circular cylinder was investigated, and also a review of the general methods of diffraction theory and a bibliography encompassing a large number of works (mainly German and American) were given.

The book of King and Yu [59] presented (as a rule without derivation) a series of asymptotic expressions relating to a slit and a circular hole and also to other diffraction objects. Here, however, equations from which one would be able to construct the scattering characteristics of a strip and a disk with any incidence of a plane wave also are missing.

Works on diffraction by three-dimensional bodies having edges are comparatively scarce. In the paper of Siegel et al. [41], the effective scattering surface for a finite cone with the incidence of a plane wave on it along the symmetry axis is calculated from elementary arguments. The expressions obtained here do not fully characterize the fringing field, and are suitable only for sharp cones to which we already referred in § 17. In the papers of Keller [44], the diffraction ray concept is used for calculating the scattering of scalar and electromagnetic plane waves by a finite circular cone with a flat base and also by a cone having a spherical rounding off instead of a flat base. The resulting expressions are not applicable in the vicinity of certain irradiation and observation directions. In § 17 we showed that the field scattered by a cone and by certain bodies of rotation is not expressed only in terms of the functions f and g, which refer to diffraction rays diverging from a wedge edge. This result evidently attests to the impossibility of complete calculation of the scattering characteristic with the diffraction ray concept.

Diffraction problems arising in antenna theory are usually distinguished by their great complexity, since the corresponding metal bodies (mirror, horn, etc.) have a complicated shape. Since the dimensions of these bodies and the dimensions of the radiating apertures are considerably larger than the wavelength, the application of physical diffraction theory to antenna problems is very promising. only the first steps have been taken in this direction. Thus, Kinber [60, 61] performed a calculation of the decoupling and lateral radiation of mirror antennas. The feature specific to mirror antennas is that diffraction rays arising at the mirror's edge undergo multiple reflection on its concave surface. This multiple reflection was studied by Kinber in more detail as applied to the concave surface of a cylinder and sphere [62, 63].

Diffraction problems relating to an antenna dipole — a thin cylindrical conductor — are investigated in Chapter VII, and references to the literature are also given there.

In conclusion, let us say a few words about diffraction of short waves by smooth bodies. The basic principles relating to such problems were set forth in the fundamental works of Fok and Leontovich. These principles were established by the following methods of mathematical diffraction theory:

1. By the method of an integral equation for the current on the surface of a good conducting body (the local character of the field in the half-shade region, see [17]);

2. By the method of asymptomtic summing of diffraction series (the current on a paraboloid [64, 65]; the propagation of radio wave: above the spherical Earth [18, 66]);

3. By the parabolic equation method (the propagation of radio waves above the flat [67] and spherical [68, 69] Earth; the field of a plane electromagnetic wave in the half-shade region for any convex body [70]).

Keller [42], basing his work on the diffraction ray concept, obtained an expression for the field in a deep shadow with diffracti by a convex cylinder with a variable curvature. In the particular cases of an elliptic and a parabolic cylinder, as was shown in the works of Vaynshteyn and Fedorov [71] and of Ivanov [35], Keller's equations agree with the results of the more rigorous mathematic investigation. This allows one to more precisely study (see [71]) the conversion of diffraction rays to ordinary rays and vice versa.

The parabolic equation method described in so-called ray coordinates is a more general approach to diffraction by convex bodies. This method allows one to obtain a general expression for the Green function in the case of a circular cylinder [72, 73]. Evidently this method can subsequently be successfully applied also to other cases, among them three-dimensional diffraction problems.

FOOTNOTES

Footnote (1) on page 138.	These calculations were performed under the guidance of P. S. Mikazan
Footnote (2) on page 150.	See the footnote on page 86.

CHAPTER VI

CERTAIN PHENOMENA CONNECTED WITH THE NONUNIFORM PART OF THE SURFACE CURRENT

In the previous chapters, a theoretical investigation was conducted of the field radiated by the nonuniform part of the current. In this chapter we will discuss a method for measuring this field (§ 26) and we will investigate the phenomenon of the reflected signal's depolarization (§ 27).

An experimental method for measuring the field from the nonuniform part of the current was first proposed for bodies of rotation in the paper of Ye. N. Mayzel's and the author [12]. Later it was shown that this method has a universal character, and is suitable for measuring the field from the nonuniform part of the current excited by a plane wave on any metal body [13].

§ 26. Measurement of the Field Radiated by the Nonuniform Part of the Current

Let an ideally conducting body of arbitrary shape be found in free space. A surface element of this body is shown in Figure 69.

The coordinate system was selected in such a way that its origin would lie near the body, and the source Q would be located in the plane $x = 0$. If the distance between the body and the source is a great deal larger than the body's dimensions, then the incident wave in the vicinity of the body may be investigated as a plane wave. Let us represent it in the form

$$E_x = E_{0x} e^{ik(y \sin\gamma + z\cos\gamma)}, \quad E_y = 0. \tag{26.01}$$

Here γ is the angle between the normal N to the wave front and the z axis.

Now let us place in front of the source, parallel to the radiated wave front, a polarizer P which transformed linear polarized radiation into a circularly polarized wave. Let the wave passing through the polarizer with an electric vector E_n lag in phase by 90° behind the wave with an electric vector E_τ (Figure 70). In this case, the polarizer achieves a clockwise rotation[1]. As a result, the incident wave field at the coordinate origin will equal

$$E_x = \frac{e^{i\frac{\pi}{4}}}{\sqrt{2}} E_{0x}, \quad H_x = \frac{e^{-i\frac{\pi}{4}}}{\sqrt{2}} E_{0x}. \tag{26.02}$$

The field scattered by the body may be represented in the wave zone in the following way:

$$\left.\begin{aligned} E_\varphi = -H_\vartheta &= \frac{iaE_{0x}}{2}\frac{e^{i\frac{\pi}{4}}}{\sqrt{2}}\overline{\Sigma}(\vartheta,\varphi)\frac{e^{ikR}}{R},\\ E_\vartheta = H_\varphi &= \frac{iaE_{0x}}{2}\frac{e^{-i\frac{\pi}{4}}}{\sqrt{2}}\Sigma(\vartheta,\varphi)\frac{e^{ikR}}{R}, \end{aligned}\right\} \tag{26.03}$$

where a is a certain length characterizing the body's size and $\overline{\Sigma}(\vartheta,\varphi)$ and $\Sigma(\vartheta,\varphi)$ are unknown angular functions. In the general case, the

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[1] Footnote appears on page 174.

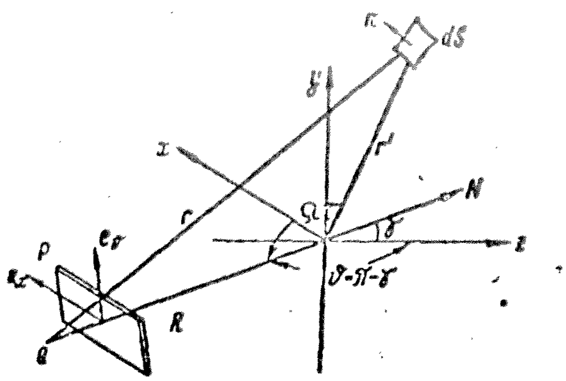

Figure 69. The problem of electromagentic wave diffraction by an arbitrary metal body.

dS - is a surface element of the body.
N - the normal to the incident wave front,
Q - the source,
P - the polarizer converting linearly polarized radiation to a wave with circular polarization.

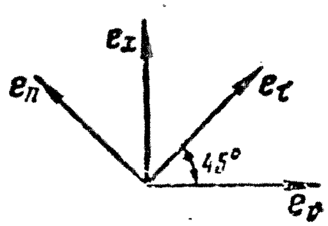

Figure 70.

field (26.03) is an elliptically polarized wave. In the direction toward the source $\left(\vartheta=\pi-\gamma,\ \varphi=-\frac{\pi}{2}\right)$ this wave passes through the polarizer and creates behind it the field

$$\left.\begin{aligned} E_x=-H_\vartheta=\frac{iaE_{0x}}{2}\sum\nolimits_{+}\frac{e^{ikR}}{R}e^{i\frac{\pi}{2}},\\ E_\vartheta=H_x=\frac{iaE_{0x}}{2}\sum\nolimits_{-}\frac{e^{ikR}}{R},\end{aligned}\right\} \tag{26.04}$$

where

$$\sum\nolimits_{\pm}=\frac{1}{2}\left(\sum\pm\overline{\sum}\right). \tag{26.05}$$

If the source radiates a wave of another polarization ($H_0 \perp yoz$), then the wave reflected by the body and passing through the polarizer is described at the point Q by similar relationships

$$\left.\begin{aligned} H_x=E_\vartheta=\frac{iaH_{0x}}{2}\sum\nolimits_{+}\frac{e^{ikR}}{R}e^{i\frac{\pi}{2}},\\ H_\vartheta=-E_x=\frac{iaH_{0x}}{2}\sum\nolimits_{-}\frac{e^{ikR}}{R}.\end{aligned}\right\} \tag{26.06}$$

Now let us investigate in the physical optics approach the diffraction of a plane linearly polarized wave by the same body. According to definition (3.01), the uniform part of the current excited on the body's surface by a plane wave with E-polarization of the incident wave ($E_0 \perp yoz$) equals

$$\left.\begin{aligned} j_x^0 &= -\frac{c}{2\pi} E_{0x} (n_y \sin\gamma + n_z \cos\gamma) e^{i\psi}, \\ j_y^0 &= \frac{c}{2\pi} E_{0x} \cdot n_x \cdot \sin\gamma \cdot e^{i\psi}, \\ j_z^0 &= \frac{c}{2\pi} E_{0x} \cdot n_x \cos\gamma e^{i\psi}, \end{aligned}\right\} \tag{26.07}$$

and with H-polarization ($H_0 \perp y0z$)

$$\left.\begin{aligned} j_x^0 &= 0, \\ j_y^0 &= \frac{c}{2\pi} H_{0x} \cdot n_z e^{i\psi}, \\ j_z^0 &= -\frac{c}{2\pi} H_{0x} \cdot n_y \cdot e^{i\psi}. \end{aligned}\right\} \tag{26.08}$$

Here E_{ox} and H_{ox} are the electric and magnetic field amplitudes of the incident wave with E-polarization and H-polarization, respectively $\psi = k(y' \sin\gamma + z' \cos\gamma)$ is the incident wave phase at the point (x', y', z') on the body's surface; n_x, n_y, n_z, are the components of the normal to the surface at the same point.

Furthermore, calculating the vector potential in the far zone on the basis of this current and substituting its values into the equations

$$\left.\begin{aligned} E_\varphi &= -H_\vartheta = ikA_\varphi, \\ E_\vartheta &= H_\varphi = ikA_\vartheta, \end{aligned}\right\} \tag{26.09}$$

we find the fringing field. With E-polarization, it equals

$$E_\varphi = -H_\vartheta = \frac{ik}{2\pi} E_{0x} \cdot \frac{e^{ikR}}{R} \cdot \int [n_x \sin\gamma \cos\varphi + \\ + (n_y \sin\gamma + n_z \cos\gamma) \sin\varphi] e^{i\Phi} dS, \tag{26.10}$$

$$E_\vartheta = H_\varphi = \frac{ik}{2\pi} E_{0x} \cdot \frac{e^{ikR}}{R} \cdot \int [n_x (\sin\gamma \cos\vartheta \sin\varphi - \cos\gamma \sin\vartheta) - \\ - (n_y \sin\gamma + n_z \cos\gamma) \cos\varphi \cos\vartheta] e^{i\Phi} dS, \tag{26.10}$$

and with H-polarization

$$E_\varphi=-H_\vartheta=\frac{ik}{2\pi}H_{0x}\cos\varphi\frac{e^{ikR}}{R}\int n_z e^{i\Phi}dS,$$

$$E_\vartheta=H_\varphi=\frac{ik}{2\pi}H_{0x}\frac{e^{ikR}}{R}\int(n_y\sin\vartheta+n_z\sin\varphi\cos\vartheta)e^{i\Phi}dS. \tag{26.11}$$

Here R, ϑ, ϕ are the spherical coordinates of the observation point, $\Phi=\psi-kr'\cos\Omega$, and integration is carried out over the illuminated elements of the body's surface. In the case of radar when the observation and irradiation directions coincide $\left(\vartheta=\pi-\gamma,\ \varphi=-\frac{\pi}{2}\right)$, Equations (26.10) and (26.11) yield

$$\left.\begin{aligned}E_x=-H_\vartheta=-\frac{ik}{2\pi}E_{0x}\frac{e^{ikR}}{R}\int(n_y\sin\gamma+\\+n_z\cos\gamma)e^{i\Phi}dS,\\E_\vartheta=H_x=0\end{aligned}\right\} \tag{26.12}$$

and

$$\left.\begin{aligned}E_\vartheta=H_x=\frac{ik}{2\pi}H_{0x}\frac{e^{ikR}}{R}\int(n_y\sin\gamma+\\+n_z\cos\gamma)e^{i\Phi}dS,\\E_x=H_\vartheta=0.\end{aligned}\right\} \tag{26.13}$$

Furthermore, assuming the incident wave amplitudes are specified by Equation (26.02), let us write Expressions (26.12) and (26.13) in the following way:

$$\left.\begin{aligned}E_x=-H_\vartheta=\frac{iaE_{0x}}{2}\frac{e^{i\frac{\pi}{4}}}{\sqrt{2}}\frac{e^{ikR}}{R}\bar{\Sigma}^0,\\E_\vartheta=H_x=\frac{iaE_{0x}}{2}\frac{e^{-i\frac{\pi}{4}}}{\sqrt{2}}\frac{e^{ikR}}{R}\Sigma^0,\end{aligned}\right\} \tag{26.14}$$

where

$$\Sigma^0=-\bar{\Sigma}^0=\frac{k}{\pi a}\int(n_y\sin\gamma+n_z\cos\gamma)e^{i\Phi}dS. \tag{26.15}$$

Now let us represent the angular functions of fringing field (26.03) in the form

$$\left.\begin{aligned}\overline{\Sigma} &= \overline{\Sigma}^0 + \overline{\Sigma}^1,\\ \Sigma &= \Sigma^0 + \Sigma^1,\end{aligned}\right\} \tag{26.16}$$

where the functions Σ^0, $\overline{\Sigma}^0$ and Σ^1, $\overline{\Sigma}^1$ refer to the field radiated by the uniform and nonuniform part of the current, respectively. Substituting these expressions into Equations (26.04) and (26.06) and taking into account relationship (26.15), let us find the fringing field passing through the polarizer P toward the source Q. In the case of E-polarization, it equals

$$\left.\begin{aligned}E_x &= -H_\vartheta = \frac{iaE_{0x}}{4}(\Sigma^1 + \overline{\Sigma}^1)\frac{e^{ikR}}{R}e^{i\frac{\pi}{2}},\\ E_\vartheta &= H_x = \frac{iaE_{0x}}{4}(2\Sigma^0 + \Sigma^1 - \overline{\Sigma}^1)\frac{e^{ikR}}{R},\end{aligned}\right\} \tag{26.17}$$

and in the case of H-polarization

$$\left.\begin{aligned}H_x &= E_\vartheta = \frac{iaH_{0x}}{4}(\Sigma^1 + \overline{\Sigma}^1)\frac{e^{ikR}}{R}e^{i\frac{\pi}{2}},\\ H_\vartheta &= -E_x = \frac{iaH_{0x}}{4}(2\Sigma^0 + \Sigma^1 - \overline{\Sigma}^1)\frac{e^{ikR}}{R}.\end{aligned}\right\} \tag{26.18}$$

The physical meaning of the result obtained is as follows. The field scattered by the body at the point Q is the sum of two waves polarized in mutually perpendicular directions. The reflected wave which is polarized the same as the primary radiation of the source is determined by the function $\Sigma_+ = \frac{1}{2}(\Sigma^1 + \overline{\Sigma}^1)$, and is created only by the nonuniform part of the current . The reflected wave with the perpendicular polarization is described by the function $\Sigma_- = 2\Sigma^0 + \Sigma^1 - \overline{\Sigma}^1$, and is the field radiated by both parts of the current. Let us note that in the general case the functions Σ^1 and $\overline{\Sigma}^1$ do not coincide, and therefore they are not balanced out in the expressions for Σ_-. In other words, the field radiated by the uniform part of the current in this case may not be separated from the fringing field.

Thus, the investigated method allows one to separate from the total field scattered by any metal body of finite dimensions that part of the field which is caused by a distortion of the surface (the curvature, a sharp bend, a point, a bulge, a hole, etc.). One should note that, in the case of electromagnetic wave scattering by a system of separate bodies, the separable part of the field is due not only to the surface's distortion, but also to the diffraction interaction of the bodies.

It is necessary, however, to keep in mind that it is possible to realize the indicated fringing field distribution not in an arbitrary observation direction, but only in a direction for which the condition $\Sigma^0 = -\overline{\Sigma}^0$ is fulfilled — for example, in the direction towards the source.

Consideration of the nonuniform part of the current also enables one to explain the reflected wave depolarization which we will investigate in the following section.

Figure 71 presents the results of measurements[(2)] and calculation of the effective scattering surface

$$\sigma^+ = \pi a^2 |\Sigma_+|^2 = \frac{1}{4}\pi a^2 |\Sigma^1 + \overline{\Sigma}^1|^2, \tag{26.19}$$

which is dependent upon the nonuniform part of the current excited by a plane electromagnetic wave on a disk. The disk's diameter equals $2a = \frac{5\lambda}{\pi}$ (λ is the wavelength). The calculations were performed with consideration of the secondary diffraction on the basis of the approximation equations for the functions Σ and $\overline{\Sigma}$ which were derived in § 24. Since it is difficult to prepare a thin disk with a sufficiently flat surface, the measurements were performed with an obtuse cone close to the shape of a disk and having a height approximately equal to one tenth of the diameter.

As is seen from Figure 71, the theoretical and experimental curves are fairly close together. A certain divergence between them, especially in the region of γ values close to 90°, may evidently be explained both by the model's conical shape and also by the

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(2) Footnote appears on page 174.

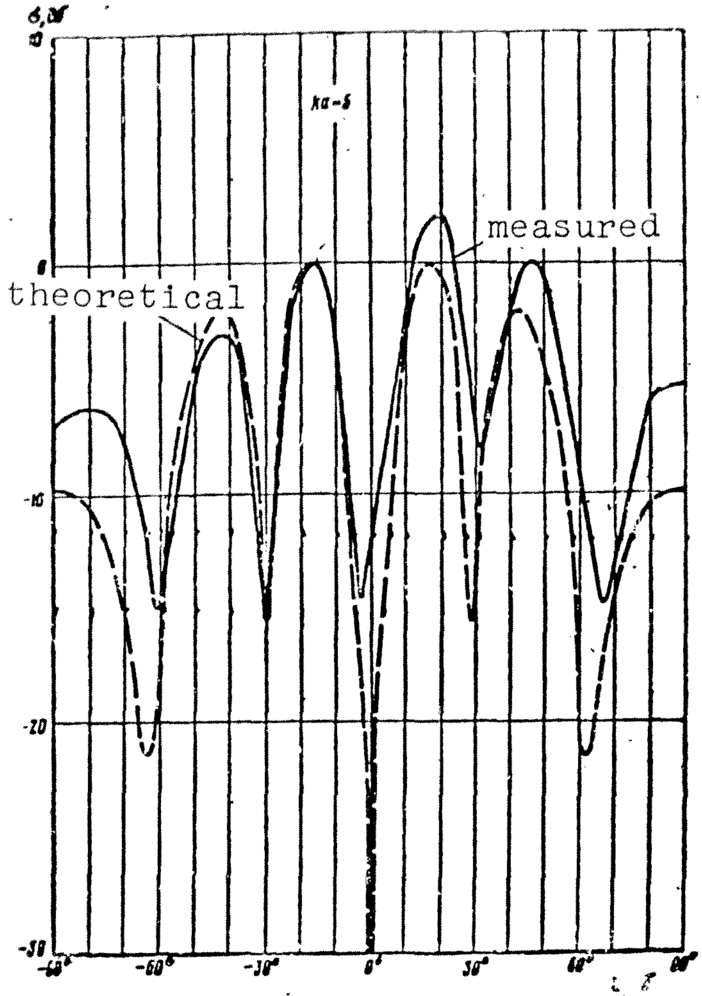

Figure 71. Diagram of the radiation from the nonuniform part of the current flowing on a disk.

approximation character of the computational equations. The value $\gamma = 90°$ corresponds to the direction along the disk's surface, and the value $\gamma = 0°$ — to the direction normal to the disk.

§ 27. Reflected Wave Depolarization

Let us again return to the problem of scattering of an electromagnetic wave by an arbitrary metal body. The relative position of the source Q, of a surface element of the irradiated body, and of the coordinate system is shown in Figure 69. Let us recall that the

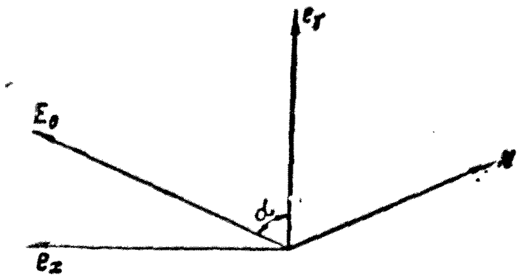

Figure 72.

source Q is in the plane y0z , and radiates a linearly polarized wave. Furthermore, we shall assume that the polarizer P which is shown in Figure 69 is now absent.

Let us designate by α the angle between the plane y0z and the incident wave electric vector E_0 (Figure 72). The field of this wave will be represented in the form

$$\begin{aligned} E_x = H_{0\gamma} = E_{0x} e^{ik(y \sin\gamma + z\cos\gamma)}, \\ H_x = -E_{0\gamma} = H_{0x} e^{ik(y \sin\alpha + z\cos\gamma)}, \end{aligned} \tag{27.01}$$

where

$$E_{0x} = E_0 \sin\alpha, \quad H_{0x} = -E_0\cos\alpha, \quad \frac{E_{0x}}{E_{0\gamma}} = \operatorname{tg}\alpha. \tag{27.02}$$

The field scattered by the body is determined in the wave zone by the equations

$$\left.\begin{aligned} E_\varphi = -H_\vartheta = \frac{ia}{2}[E_{0x}\bar{\Sigma}_1(\gamma,\vartheta,\varphi) + \\ + H_{0x}\Sigma_1(\gamma,\vartheta,\varphi)]\frac{e^{ikR}}{R}, \\ E_\vartheta = H_\varphi = \frac{ia}{2}[E_{0x}\bar{\Sigma}_2(\gamma,\vartheta,\varphi) + \\ + H_{0x}\Sigma_2(\gamma,\vartheta,\varphi)]\frac{e^{ikR}}{R}. \end{aligned}\right\} \tag{27.03}$$

Here a is a certain length characterizing the body's dimensions, R, ϑ , ϕ are the spherical coordinates of the observation point, $\bar{\Sigma}_{1,2}(\gamma,\vartheta,\varphi)$ and $\Sigma_{1,2}(\gamma,\vartheta,\varphi)$ are unknown angular functions.

It is obvious that the fringing field polarization — that is, the orientation of its electric vector in space — depends in a complex way on the observation and irradiation directions. In the direction toward the source, it may not coincide with the polarization of the wave radiated by the source. Such a phenomenon is called reflected wave depolarization.

It is easy to establish the reason for depolarization, if one investigates the fringing field as the sum of the fields radiated by the uniform and nonuniform parts of the current. According to § 26, the uniform part of the current radiates the following field in the direction towards the source $\left(\vartheta=\pi-\gamma,\ \varphi=-\frac{\pi}{2}\right)$

$$\left.\begin{aligned} E_x = -H_\vartheta = \frac{iaE_{0x}}{2}\frac{e^{ikR}}{R}\bar{\Sigma}^0, \\ E_\vartheta = H_\varphi = \frac{iaH_{0x}}{2}\frac{e^{ikR}}{R}\Sigma^0. \end{aligned}\right\} \tag{27.04}$$

The functions $\bar{\Sigma}^0$ and Σ^0 satisfy the condition $\Sigma^0 = -\bar{\Sigma}^0$, and are described by Equation (26.15). From Equation (27.04), let us immediatel obtain the equality

$$\frac{E_x}{E_\vartheta} = \operatorname{tg}\alpha, \tag{27.05}$$

which means that in the physical optics approach the reflected wave does not experience depolarization. Consequently, the reflected wave depolarization is caused only by the nonuniform part of the current or, in other words, by the surface distortion.

Let us derive an equation for the magnitude of angle δ. This is the angle by which the electric field vector of the reflected wave is turned in respect to the electric vector of the wave radiated by the source. For this purpose, let us represent the functions $\bar{\Sigma}_{1(2)}$ and $\Sigma_{1(2)}$ in the form

$$\left.\begin{aligned} \bar{\Sigma}_{1(2)} = \bar{\Sigma}^0_{1(2)} + \bar{\Sigma}^1_{1(2)}, \\ \Sigma_{1(2)} = \Sigma^0_{1(2)} + \Sigma^1_{1(2)}, \end{aligned}\right\} \tag{27.06}$$

where the terms $\bar{\Sigma}^0_{1(2)}$ and $\Sigma^0_{1(2)}$ correspond to the field radiated by the uniform part of the current, and the terms $\bar{\Sigma}^1_{1(2)}$ and $\Sigma^1_{1(2)}$

correspond to the field radiated by the nonuniform part of the current Comparing Expressions (27.04) and (27.03), we find that

$$\left.\begin{aligned} \bar{\Sigma}_1^0 &= \bar{\Sigma}^0, & \Sigma_1^0 &= 0, \\ \bar{\Sigma}_2^0 &= 0, & \Sigma_2^0 &= -\bar{\Sigma}^0. \end{aligned}\right\} \tag{27.07}$$

Therefore, the field scattered in the direction towards the source $\left(\vartheta = \pi - \gamma,\ \varphi = -\frac{\pi}{2}\right)$, will equal

$$\left.\begin{aligned} E_x &= -H_\vartheta = \frac{ia}{2}\left[E_{0x}\left(\bar{\Sigma}^0 + \bar{\Sigma}_1^1\right) + H_{0x}\Sigma_1^1\right]\frac{e^{ikR}}{R}, \\ E_\vartheta &= H_x = \frac{ia}{2}\left[E_{0x}\bar{\Sigma}_2^1 - H_{0x}\left(\bar{\Sigma}^0 - \Sigma_2^1\right)\right]\frac{e^{ikR}}{R}. \end{aligned}\right\} \tag{27.08}$$

This field's electric vector forms an angle β with the yoz plane. The angle β is determined by the equation

$$\operatorname{tg}\beta = \frac{E_x}{E_\vartheta} = \frac{\bar{\Sigma}^0 + \bar{\Sigma}_1^1 - \Sigma_1^1 \operatorname{ctg}\alpha}{\bar{\Sigma}^0 - \Sigma_2^1 + \bar{\Sigma}_2^1 \operatorname{tg}\alpha}\operatorname{tg}\alpha. \tag{27.09}$$

As a result, the desired angle δ which characterizes the depolarizatic magnitude will equal

$$\delta = \alpha - \beta. \tag{27.10}$$

Thus, the field from the nonuniform part of the current, separabl "in a pure form" by means of a polarizer (§ 26), leads to depolarizat of the scattered radiation.

Specific results from the depolarization calculation of waves reflected from certain bodies may be found, for example, in the works of Chytil [75 - 77] and Beckmann [78]. In particular, in Reference [' it was shown that the depolarization effect on the effective scattering surface of convex bodies in practice may be neglected only with the condition $ka \gtrsim 4$.

FOOTNOTES

Footnote (1) on page 164. A system of metal plates parallel to the e_{τ} vector may serve as the simplest example of such a polarizer.

Footnote (2) on page 169. See the footnote on page 86.

CHAPTER VII

DIFFRACTION BY A THIN CYLINDRICAL CONDUCTOR

In almost all the works devoted to the diffraction of plane electromagnetic waves by a thin cylindrical conductor, the current induced in the conductor was studied, and then, by integrating this current, the fringing field in the far zone was calculated. However, in view of the complexity of this problem, they succeeded in obtaining relatively simple equations only in the particular case when the observation direction and the direction toward the source coincided, and was perpendicular to the conductor axis. In the general case when these directions did not coincide and were arbitrary, the expressions for the fringing field became very complicated and unsuitable for making calculations. Since they were obtained by integrating approximation expressions for the current, it turns out that they have still one other shortcoming — they do not satisfy the principle of duality.

In this chapter, explicit expressions are obtained for the fringing field which are suitable for making calculations with any direction of irradiation and observation. We shall consider both the primary edge waves excited by the incident plane wave and also the secondary, tertiary, etc. edge waves. The total fringing field is found by summing all the diffraction waves.

§ 28. Current Waves in an Ideally Conducting Vibrator

The electrodynamic problem of determining the current in thin cylindrical conductors (vibrator) usually is reduced to an integro-differential equation. The latter is derived by means of boundary conditions on the conductor surface, and is substantially simplified in the case of thin conductors when the inequalities

$$\frac{a}{L} \ll 1 \text{ and } ka \ll 1, \tag{28.01}$$

are fulfilled, where a is the radius and L is the length of the conductor and $k=\frac{2\pi}{\lambda}=\frac{\omega}{c}$.

Its solution may be found, for example, by the method of successive approximations [79, 80] or by the perturbation method [85]. Recently, Vaynshteyn [81, 82] proposed a new solution for this equatic Since we will subsequently base our work on the results of References [81, 82], let us discuss them in more detail.

Let us assume that the vibrator's symmetry axis coincides with the z axis, and its ends have the coordinates $z = z_1$ and $z = z_2$ ($L = z_2 - z_1$). In the case of excitation of the dipole by a concentrated external field

$$E_z^e = \mathcal{E}\delta(z) \tag{28.02}$$

the current J(z) in the conductor may obviously be written in the forı of the sum of the waves travelling along the conductor with a velocit; c from the excitation point z = 0 and the ends $z = z_1$ and $z = z_2$. In Reference [81] it was shown that the complex amplitudes of these wave: are slowly varying functions of the z coordinate. These functions may be approximately expressed in terms of the function $\psi(z)$, so that we obtain the following expression for the current J(z):

$$J(z)=J_0[\psi(|z|)\,e^{ik|z|}+A_1\psi(z-z_1)e^{ik(z-z_1)}+ \\ +A_2\psi(z_2-z)\,e^{ik_2(z_2-z)}]. \tag{28.03}$$

Here the quantity

$$J_0 = \frac{c\mathcal{E}}{4\ln\frac{i}{\gamma ka}}, \ \gamma = 1{,}781\ldots \tag{28.04}$$

determines the initial value of the current wave propagated from the excitation point[1]. The function $\psi(z)$ is the solution of the integral equation, and in addition to the variable z it also depends on the parameters k and a. We will not list here all the properties of the function $\psi(z)$, but let us note only that it satisfies the conditions

$$\psi(0) = 1, \quad \psi(\infty) = 0, \tag{28.05}$$

and its absolute value monotonically decreases with an increase of z. This decrease, which is rather slow and does not have an exponential character, is due to radiation.

The constants A_1 and A_2 determine the initial values of the current waves originating at the points $z = z_1$ and $z = z_2$, respective: and travelling in the direction towards the opposite end of the conductor. These constants wre found from the conditions at the conductor ends

$$J(z_1) = J(z_2) = 0 \tag{28.06}$$

and equal

$$\left.\begin{aligned} A_1 &= -\frac{1}{D}\left[\psi(-z_1) - \psi(z_2)\psi(L)\,e^{2ikz_2}\right]e^{-ikz_1}, \\ A_2 &= -\frac{1}{D}\left[\psi(z_2) - \psi(-z_1)\psi(L)\,e^{-2ikz_1}\right]e^{ikz_2}, \end{aligned}\right\} \tag{28.07}$$

where

$$D = 1 - \psi^2(L)\,e^{2ikL}. \tag{28.08}$$

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[1]Footnote appears on page 216.

Considering that the quantity 1/D is equal to the infinite geometric progression,

$$\frac{1}{D} = 1 + \psi^2(L)e^{2ikL} + \psi^4(L)e^{4ikL} + \ldots, \tag{28.09}$$

Expression (28.03) may be written in the expanded form

$$\begin{aligned} J(z) = J_0\{&\psi(|z|)e^{ik|z|} - \psi(-z_1)e^{-ikz_1}[\psi(z-z_1)e^{ik(z-z_1)} - \\ &- \psi(L)e^{ikL}\psi(z_2-z)e^{ik(z_2-z)} + \psi^2(L)e^{2ikL}\psi(z - \\ &- z_1)e^{ik(z-z_1)} - \ldots] - \psi(z_2)e^{ikz_2}[\psi(z_2-z)e^{ik(z_2-z)} - \\ &- \psi(L)e^{ikL}\psi(z-z_1)e^{ik(z-z_1)} + \\ &+ \psi^2(L)e^{2ikL}\psi(z_2-z)e^{ik(z_2-z)} - \ldots]\}. \end{aligned} \tag{28.10}$$

The physical meaning of Expression (28.03) is seen from this. The first term in Equation (28.10) is the primary current wave which coincides with the wave excited by a concentrated emf in an infinitely long conductor. The second term (in all brackets) corresponds to the current resulting from the reflection of the primary current wave from the conductor end $z = z_1$, and as a result of subsequent reflections from the conductor ends which arise from this wave $-J_0\psi(-z_1)e^{-ikz_1}\psi(z-z_1)e^{ik(z-z_1)}$. The third term (in all brackets) corresponds to the current resulting from the reflection of the primary wave from the end $z = z_2$ and as a result of the subsequent reflections from the conductor ends arising from this wave $-J_0\psi(z_2)e^{ikz_2}\psi(z_2-z)e^{ik(z_2-z)}$.

It also follows from Equation (28.03) that external field (28.02 excites in the semi-infinite conductor $(z_1 \leqslant z \leqslant \infty)$ the current

$$J(z) = J_0\{\psi(|z|)e^{ik|z|} - \psi(-z_1)e^{-ikz_1}\psi(z-z_1)e^{ik(z-z_1)}\}, \tag{28.11}$$

and in the semi-infinite conductor $(-\infty \leqslant z \leqslant z_2)$ the current

$$J(z) = J_0[\psi(|z|)e^{ik|z|} - \psi(z_2)e^{ikz_2}\psi(z_2-z)e^{ik(z_2-z)}]. \tag{28.12}$$

Comparing these expressions with the proper terms in (28.10), we see that the reflection of all the current waves at the end of a finite length vibrator occurs in the same way as at the end of a semi-infinite conductor.

In the case of a passive vibrator $(z_1 \leq z \leq z_2)$, excited by the plane wave

$$E_z^e = E_{0z} e^{iwz}, \quad w = -k\cos\vartheta, \tag{28.13}$$

the current also is represented in the form of the sum of waves (see [82])

$$\begin{aligned} J(z) = S[e^{iwz} - \psi_-(z - z_1)\, e^{iwz_1 + ik(z-z_1)} - \\ - \psi_+(z_2 - z)\, e^{iwz_2 + ik(z_2 - z)} + \bar{A}_1 \psi(z - z_1)\, e^{ik(z-z_1)} + \\ + \bar{A}_2 \psi(z_2 - z)\, e^{ik(z_2 - z)}], \end{aligned} \tag{28.14}$$

where the first term corresponds to the current excited by a plane wave in an infinitely long conductor. Its complex amplitude S equals

$$S = \frac{i\omega E_{0z}}{2k^2 \sin^2\vartheta \ln \frac{2i}{\gamma ka \sin\vartheta}}. \tag{28.15}$$

The second and third terms are primary edge waves arising as a consequence of the cut-off of the current Se^{iwz}. They are expressed in terms of the functions $\psi_+(z)$ and $\psi_-(z)$ which depend, in addition to the variable z and the parameters k and a, on the angle ϑ. These functions satisfy the relationships

$$\left.\begin{aligned} &\psi_\pm(0) = 1, \quad \psi_\pm(\infty) = 0, \\ &\psi_+(z)\big|_{\vartheta=\pi} = \psi_-(z)\big|_{\vartheta=0} = \psi(z), \\ &\psi_+(z)\big|_{\vartheta=0} = \psi_-(z)\big|_{\vartheta=\pi} = 1. \end{aligned}\right\} \tag{28.16}$$

The initial values of the primary edge waves are such that their sum with the wave Se^{iwz} gives a current equal to zero at the conductor ends.

The last two terms in Equation (28.14) correspond to secondary, tertiary, etc., edge waves, and have the same form as they do for a transmitting vibrator [compare Equation (28.03)]. The unknown coefficients $\tilde{A}_1$ and $\tilde{A}_2$ are found from Conditions (28.06) and equal

$$\left.\begin{aligned} \tilde{A}_1 &= \frac{1}{D} e^{i(k+w)z_2} [\psi_+(L) - \\ &- \psi_-(L)\psi(L) e^{i(k-w)L}] e^{-ikz_1}, \\ \tilde{A}_2 &= \frac{1}{D} e^{-i(k-w)z_1} [\psi_-(L) - \\ &- \psi_+(L)\psi(L) e^{i(k+w)L}] e^{ikz_2}. \end{aligned}\right\} \tag{28.17}$$

Using equality (28.09), Expression (28.14) may be written in the more graphic form

$$\left.\begin{aligned} J(z) = S\{ & e^{iwz} - \psi_-(z-z_1) e^{iwz_1+ik(z-z_1)} + \\ & + \psi_-(L) e^{iwz_1+ikL} [\psi(z_2-z) e^{ik(z_2-z)} - \\ & - \psi(L) e^{ikL} \psi(z-z_1) e^{ik(z-z_1)} + \\ & + \psi^2(L) e^{2ikL} \psi(z_2-z) e^{ik(z_2-z)} - \ldots] - \\ & - \psi_+(z_2-z) e^{iwz_2+ik(z_2-z)} + \\ & + \psi_+(L) e^{iwz_2+ikL} [\psi(z-z_1) e^{ik(z-z_1)} - \\ & - \psi(L) e^{ikL} \psi(z_2-z) e^{ik(z_2-z)} + \\ & + \psi^2(L) e^{2ikL} \psi(z-z_1) e^{ik(z-z_1)} - \ldots]\}. \end{aligned}\right\} \tag{28.18}$$

Here besides the wave Se^{iwz} and the primary edge waves, which we talked about in connection with Equation (28.14), the secondary, tertiary, etc. waves diverging from the ends $z = z_1$ and $z = z_2$ are explicitly written out; they correspond to the first, second, etc. terms in the graphs.

Passing to the limit in Equation (28.14) when $z_2 \to \infty$, we find the current in the semi-infinite conductor (z_1, ∞)

$$J(z) = S \cdot [e^{iwz} - \psi_-(z-z_1) e^{iwz_1+ik(z-z_1)}] \tag{28.19}$$

and, similarly, we find the current in the semi-infinite conductor $(-\infty, z_2)$

$$J(z) = S \cdot [e^{iwz} - \psi_+(z_2-z) e^{iwz_2+ik(z_2-z)}]. \tag{28.20}$$

It is not difficult to see that in the case of a passive vibrator the reflection of current waves at its ends occurs in the same way as at the end of a semi-infinite conductor.

Thus, the complex amplitudes of current waves in a thin, finite length conductor are proportional to the functions $\psi(z)$ and $\psi_{\pm}(z)$ which monotonically decrease with an increase of z as a consequence of radiation. Let us note several properties of current waves in a vibrator. Each advancing wave in sum with the reflected wave excited by it gives a zero current at the conductor's end. In the case $L = z_2 - z_1 \approx n\frac{\lambda}{2}$ $(n = 1, 2. 3 \ldots)$ and $D \approx 0$, a current resonance begins in the vibrator.

The precision of Expressions (28.03) and (28.14) obtained by the method of slowly varying functions is different in various sections of the conductor. It is comparatively low near the conductor's ends (and in the vicinity of the point z = 0 of a transmitting vibrator) where the current waves arise, and where their complex amplitude varie rather rapidly. As the distance from these vibrator elements increase the precision of these equations increases without limit.

It should be stated that with a more rigorous approach [79, 90] the amplitudes of all the reflected waves will be determined by different functions; however, the difference between them rapidly decreas with an increase of the reflection number. The functions $\psi(z)$ and $\psi_{\pm}(z)$ only approximately describe these current waves, but on the othe hand they allow one to effectively sum them and to obtain closed equations.

Using the variational method for the functions $\psi(z)$ and $\psi_{\pm}(z)$, we obtained the approximate, but on the other hand, simple equations (see [83])

$$\psi(z) = \frac{\ln\frac{-1}{\gamma^2 q}}{\ln\frac{2ix}{\gamma} - E(2qx)e^{-2iqx}}, \qquad (28.21)$$

(Equation continued on next page)

$$\psi_{\pm}(z)=\frac{\ln\frac{-1}{\gamma^2 q_{\pm}}}{\ln\frac{2ix}{\gamma}-E(2q_{\pm}x)e^{-2iq_{\pm}x}}, \tag{28.21}$$

where

$$\ln(-1)=i\pi,\quad \ln i=i\frac{\pi}{2}, \tag{28.22}$$

$$x=\frac{kz}{q},\quad q=(ka)^2,\quad q_{\pm}=q\frac{1\mp\cos\vartheta}{2} \tag{28.23}$$

and

$$E(y)=\mathrm{Ci}\,y+i\,\mathrm{si}\,y=-\int_y^{\infty}\frac{e^{it}}{t}\,dt. \tag{28.24}$$

The integral cosine Ci y and the integral sine si y are determined by the relationships

$$\mathrm{Ci}\,y=-\int_y^{\infty}\frac{\cos t}{t}\,dt,\quad \mathrm{si}\,y=-\int_y^{\infty}\frac{\sin t}{t}\,dt \tag{28.25}$$

and are thoroughly tabulated functions.

The equations written above for the current in a finite conductor are distinguishable by their visualizability, and they enable one to liken the conductor to a section of a transmission line in which, however, the attenuation of the current waves takes place, not according to an exponential law, but according to a more complicated law which is determined by Equations (28.21). In addition, the diffraction character of the problem is reflected in the equations. The conductor's specific features as a diffraction object are included in the very slow attenuation of the current waves. As a consequence of this, it is impossible to limit oneself to considering only secondary and tertiary diffractions, and it is necessary to sum all the reflected waves. As a result of such a summation, a "resonance denominator" D appears which takes into account the resonance properties of a thin, finite length conductor.

§ 29. Radiation of a Transmitting Vibrator

The radiation characteristic of a transmitting vibrator may be calculated from a known equation by integrating the currents in it. However, such an approach is not advisable because, as was indicated above, the precision of Equations (28.03) is different in different parts of the conductor, and is low near its ends ($z = z_1$ and $z = z_2$) and the point $z = 0$. The principle of duality gives more precise results. This principle leads to the following expression for the radiation field in the far zone [82]:

$$\left.\begin{aligned} E_\vartheta = H_\varphi &= \frac{\mathcal{E}}{2\sin\vartheta \ln\dfrac{2i}{\gamma k a \sin\vartheta}} \frac{e^{ikR}}{R}\cdot f(\vartheta), \\ E_\varphi = H_\vartheta &= 0. \end{aligned}\right\} \tag{29.01}$$

The function

$$\begin{aligned} f(\vartheta) = 1 &- \psi_+(z_2)\, e^{ikz_2(1-\cos\vartheta)} - \psi_-(-z_1)\, e^{-ikz_1(1+\cos\vartheta)} + \\ &+ B_1\psi_+(L)\, e^{ikz_2(1-\cos\vartheta)} + B_2\psi_-(L)\, e^{-ikz_1(1+\cos\vartheta)} \quad (29.02) \end{aligned} \tag{29.02}$$

is connected with the current (28.14) excited in a vibrator by plane wave (28.13) by the relationship

$$J(0) = Sf(\vartheta), \tag{29.03}$$

The coefficients B_1 and B_2 do not depend on the angle ϑ.

Expression (29.02) enables one to trace the formation of the radiation. The first term (one) is the radiation field of an infinitely long conductor excited by a concentrated emf. Propagating in the direction $\vartheta = 0$, this field reaches the conductor's end $z = z_2$ and — being diffracted by it — generates a primary edge wave (the second term). In a similar way, the primary edge wave diverging from the conductor's end $z = z_1$ is excited (the third term). The last two terms in Equation (29.02) determine the waves arising as a result of subsequent diffraction (secondary, tertiary, etc.). The amplitudes B_1 and B_2 of these waves may be found from the conditions

$$f(0)=f(\pi)=0, \tag{29.04}$$

which means that the radiation of a finite conductor in the direction of its geometric extension must be equal to zero. These conditions, together with a consideration of the relationships (28.05) and (28.16), lead to the system of equations.

$$\left.\begin{aligned} B_1+B_2\psi(L)\,e^{-2ikz_1}&=\psi(-z_1)\,e^{-2ikz_1},\\ B_1\psi(L)\,e^{2ikz_2}+B_2&=\psi(z_2)\,e^{2ikz_2},\end{aligned}\right\} \tag{29.05}$$

from which we find without difficulty

$$\left.\begin{aligned} B_1&=\frac{1}{D}\{\psi(-z_1)-\psi(L)\psi(z_2)\,e^{2ikz_2}\}\,e^{-2ikz_1},\\ B_2&=\frac{1}{D}\{\psi(z_2)-\psi(L)\psi(-z_1)\,e^{-2ikz_1}\}\,e^{2ikz_2}.\end{aligned}\right\} \tag{29.06}$$

Keeping in mind (28.09), let us represent the functions f(ϑ) in a more graphic form

$$\begin{aligned} f(\vartheta)=1&-\psi_+(z_2)\,e^{ikz_2(1-\cos\vartheta)}+\psi(z_2)\,e^{ik(L+z_1)}\times\\ &\times[\psi_-(L)\,e^{-ikz_1\cos\vartheta}-\psi(L)\,e^{ikL}\psi_+(L)\,e^{-ikz_1\cos\vartheta}+\\ &+\psi^2(L)\,e^{2ikL}\psi_-(L)\,e^{-ikz_1\cos\vartheta}-\ldots]-\\ &-\psi_-(-z_1)\,e^{-ikz_1(1+\cos\vartheta)}+\psi(-z_1)\,e^{ik(L-z_1)}\times\\ &\times[\psi_+(L)\,e^{-ikz_1\cos\vartheta}-\psi(L)\,e^{ikL}\cdot\psi_-(L)\,e^{-ikz_1\cos\vartheta}+\\ &+\psi^2(L)\,e^{2ikL}\cdot\psi_+(L)\,e^{-ikz_1\cos\vartheta}-\ldots],\end{aligned} \tag{29.07}$$

where the secondary, tertiary, etc. waves corresponding to the first, second, and following terms in the brackets are explicitly written out.

Thus, the field radiated by a transmitting vibrator arises as a result of multiple diffraction of edge waves at the vibrator's ends. Let us note in connection with this that the edge wave is diffracted by the opposite end of the vibrator in the same way as at the end of a corresponding semi-infinite conductor. It is not difficult to establish this by investigating the radiation of a semi-infinite conductor excited by a concentrated emf.

§ 30. Primary and Secondary Diffraction by a Passive Vibrator

Let a plane electromagnetic wave fall at an angle ϑ_0 on a thin cylindrical conductor of length $L = z_2 - z_1$ and radius a (Figure 73). For purposes of generality, we will consider that the incident wave's electric field E_0 forms an angle α with the plane of the figure. Then, its tangential component on the conductor surface will equal

$$E_z^e = E_{0z} \cdot e^{iw_0 z}, \tag{30.01}$$

where

$$E_{0z} = E \sin \vartheta_0, \quad E = E_0 \cos \alpha, \quad w_0 = -k \cos \vartheta_0. \tag{30.02}$$

The current induced in the vibrator by this field was investigated by us in § 28. As was already indicated above, Expression (28.14) which was obtained for it has a relatively low precision near the conductor ends. Therefore, it is inadvisable to seek the fringing field by integration of the current. Let us also note that the fringing field found by such a method does not satisfy the principle of duality.

We shall seek the scattering characteristic of a passive vibrator by starting from the following scattering picture which naturally follows from the previous results. An incident plane wave, being diffracted at the conductor ends, excites primary edge waves which are radiated into the surrounding space. Being propagated along the conductor, each of these waves experiences diffraction at the opposite end of the conductor and excites secondary edge waves. The latter, in turn, generate tertiary edge waves, etc. The total fringing field is comprised of the sum of all the edge waves being formed during sequential (multiple) diffraction at the conductor's ends.

In § 28 and 29, we noted that current waves are reflected from the ends of a finite length conductor the same as from the end of a semi-infinite conductor, and that the diffraction of these waves at each end takes place in the same way as at the end of a semi-infinite conductor. Therefore, the primary edge waves may be found from the problem of scattering of a plane wave by the semi-infinite conductor (z_1, ∞) and the conductor $(-\infty, z_2)$. The sum of such waves gives the primary diffraction field

$$E_\vartheta^{(1)} = H_\varphi^{(1)} = -E\,\frac{e^{ikR}}{kR}\cdot F^{(1)}(\vartheta, \vartheta_0), \tag{30.03}$$

where

$$\begin{aligned} F^{(1)}(\vartheta, \vartheta_0) = \\ = \frac{i}{2}\,\frac{\operatorname{ctg}\frac{\vartheta}{2}\operatorname{ctg}\frac{\vartheta_0}{2}}{(\cos\vartheta + \cos\vartheta_0)\,\Phi(-k\cos\vartheta, -k\cos\vartheta_0)}\, e^{-ikz_1(\cos\vartheta + \cos\vartheta_0)} - \\ - \frac{i}{2}\,\frac{\operatorname{tg}\frac{\vartheta}{2}\operatorname{tg}\frac{\vartheta_0}{2}}{(\cos\vartheta + \cos\vartheta_0)\,\Phi(k\cos\vartheta, k\cos\vartheta_0)}\, e^{-ikz_2(\cos\vartheta + \cos\vartheta_0)}. \end{aligned} \tag{30.04}$$

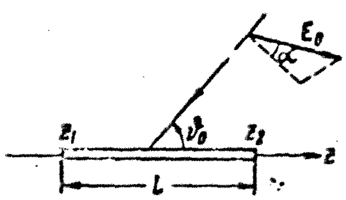

Figure 73. The incidence of a plane wave on a thin cylindrical conductor; ϑ_0 is the incident angle.

The function $\Phi(w, w_0)$ may be calculated by means of the rigorous solution to the problem of a semi-infinite vibrator (see [82] § 3 and [83] § 4), and in this case, it satisfies the relationship

$$\left.\begin{aligned} \Phi(w, w_0)\,\Phi(-w, -w_0) = \ln\frac{2i}{\gamma v a}\ln\frac{2i}{\gamma v_0 a}, \\ v = \sqrt{k^2 - w^2}, \quad v_0 = \sqrt{k^2 - w_0^2}. \end{aligned}\right\} \tag{30.05}$$

However, henceforth it will not be necessary to have the rigorous expression for the function ϕ. Let us note that Equations (30.03) and (30.04) are similar to Expressions (6.13) for a strip. These latter expressions do not take into account secondary diffraction.

The secondary edge wave propagated from the end $z = z_2$ is excited during diffraction at this end of the primary current wave

$$-S\psi_-^0(z-z_1)\,e^{i\bar{w}_0 z_1 + ik(z-z_1)}, \tag{30.06}$$

where by $\psi_\pm^0(z)$ we mean the functions obtained from the functions $\psi_\pm(z)$ by replacing ϑ by ϑ_0. For the purpose of calculating the desired secondary waves, it is necessary for us, first of all, to find that external field which, when applied to an infinite conductor $(-\infty \leqslant z \leqslant \infty)$, would excite the current (30.06) on its section $(z_1 \leqslant z \leqslant \infty)$.

For this purpose, let us study the current induced in an infinite conductor by the external field

$$E_z^e = \hat{E}_{0z} e^{i\bar{w}_0 z} \cdot \varepsilon(z-z_1), \quad \varepsilon(z-z_1) = \begin{cases} 1 \text{ with } z < z_1 \\ 0 \text{ with } z > z_1. \end{cases} \tag{30.07}$$

Let us assume that ω_0 has a small negative imaginary part (Im $\omega_0 \leq 0$). We may regard the quantity $\hat{E}_{0z} e^{i\bar{w}_0\zeta} d\zeta$ as a concentrated emf which, in accordance with equation (28.03), creates in an infinite conductor

$$\frac{c\hat{E}_{0z}}{4\ln\frac{i}{\gamma ka}}\psi(|z-\zeta|)\,e^{i\bar{w}_0\zeta + ik|z-\zeta|}\,d\zeta. \tag{30.08}$$

Therefore, in accordance with the principle of superposition, the total current created in the region $z_1 \leqslant z \leqslant \infty$ by the external field (30.07) will equal

$$J(z) = \frac{c\hat{E}_{0z}}{4\ln\frac{i}{\gamma ka}} \int_{-\infty}^{z_1} \psi(z-\zeta)\,e^{i\bar{w}_0\zeta + ik(z-\zeta)}\,d\zeta = $$
$$= \frac{c\hat{E}_{0z}}{4\ln\frac{i}{\gamma ka}} e^{i\bar{w}_0 z} \cdot \int_{z-z_1}^{\infty} \psi(\xi)\,e^{i(k-\bar{w}_0)\xi}\,d\xi. \tag{30.09}$$

The resulting integral may be expressed in terms of the functions $\psi(z - z_1)$ and $\psi_-^0(z - z_1)$, and the corresponding relationships derived in § 2 of [82]. As a result, we find

$$J(z)=\frac{c\hat{E}_{0z}e^{iw_0z_1}}{8ik\sin^2\frac{\vartheta_0}{2}\ln\frac{i}{\gamma ka}}\psi(z-z_1)e^{i(z-z_1)}-$$
$$-\frac{c\hat{E}_{0z}e^{iw_0z_1}}{2ik\sin^2\vartheta_0\ln\frac{i}{\gamma ka\cos\frac{\vartheta_0}{2}}}\psi^0_-(z-z_1)e^{ik(z-z_1)}. \qquad (30.10)$$

Thus, it turns out that external field (30.07) excites, in addition to the wave ψ^0_-, also the wave ψ. In order to excite a "pure" ψ^0_- wave, it is necessary obviously to apply an additional external field

$$E^e_z=\mathcal{E}_1\delta(z-z_1), \qquad (30.11)$$

such that

$$\frac{c\hat{E}_{0z}e^{iw_0z_1}}{8ik\sin^2\frac{\vartheta_0}{2}\ln\frac{i}{\gamma ka}}\psi(z-z_1)e^{ik(z-z_1)}+$$
$$+\frac{c\mathcal{E}_1}{4\ln\frac{i}{\gamma ka}}\psi(z-z_1)e^{ik(z-z_1)}=0. \qquad (30.12)$$

Hence,

$$\mathcal{E}_1=-\frac{\hat{E}_{0z}e^{iw_0z_1}}{2ik\sin^2\frac{\vartheta_0}{2}}. \qquad (30.13)$$

In order that the sum of external fields (30.07) and (30.11) would create the current (30.06), it is still necessary to fulfill the equality

$$\frac{c\hat{E}_{0z}}{2ik\sin^2\vartheta_0\ln\frac{i}{\gamma ka\cos\frac{\vartheta_0}{2}}}=S=\frac{i\omega E_{0z}}{2k^2\sin^2\vartheta_0\ln\frac{2i}{\gamma ka\sin\vartheta_0}}, \qquad (30.14)$$

which determines the quantity

$$\hat{E}_{0z} = -E_{0z}\frac{\ln\dfrac{i}{\gamma ka\cos\dfrac{\vartheta_0}{2}}}{\ln\dfrac{2i}{\gamma ka\sin\vartheta_0}}. \qquad (30.15)$$

Consequently, for the excitation in an infinite conductor (with $z > z_1$) of current waves (30.06), it is necessary to apply the external field

$$E_z^e = E_{0z}\frac{\ln\dfrac{i}{\gamma ka\cos\dfrac{\vartheta_0}{2}}}{\ln\dfrac{2i}{\gamma ka\sin\vartheta_0}}\left[\frac{e^{iw_0 z_1}}{2ik\sin^2\dfrac{\vartheta_0}{2}}\delta(z-z_1) - e^{iw_0 z}\varepsilon(z-z_1)\right],$$

$$\varepsilon(z-z_1) = \begin{cases} 1 \text{ with } z < z_1 \\ 0 \text{ with } z > z_1. \end{cases} \qquad (30.16)$$

In a completely similar way, one may show that the external field

$$E_z^e = E_{0z}\frac{\ln\dfrac{i}{\gamma ka\sin\dfrac{\vartheta_0}{2}}}{\ln\dfrac{2i}{\gamma ka\sin\vartheta_0}}\left[\frac{e^{iw_0 z_2}}{2ik\cos^2\dfrac{\vartheta_0}{2}}\delta(z_2-z) - e^{iw_0 z}\varepsilon(z_2-z)\right],$$

$$\varepsilon(z_2-z) = \begin{cases} 1 \text{ with } z > z_2 \\ 0 \text{ with } z < z_2 \end{cases} \qquad (30.17)$$

excites the following current in an infinite single-wire line (with $z < z_2$)

$$-Se^{iw_0 z_2}\cdot\psi_4^0(z_2-z)\,e^{ik(z_2-z)}, \qquad (30.18)$$

Now let us study the diffraction of current waves (30.06) by the semi-infinite conductor $(-\infty, z_2)$. For this purpose, let us use the Lorentz lemma [4]

$$\int\left(\mathbf{j}_1^e\mathbf{E}_2 + \mathbf{j}_2^m\mathbf{H}_1\right)dV = 0. \qquad (30.19)$$

Here $\mathbf{j}_1^e = -i\omega\mathbf{p}_1\delta(\mathbf{R}-\mathbf{R}')$ is the current of the auxiliary dipole with the moment $\mathbf{p}_1$ which is located at point 1 with the coordinates (R, ϑ); $\mathbf{H}_1$

is its field on the conductor surface, where the external currents j_2^m are specified; E_2 is the field created by these currents at point 1 (Figure 74).

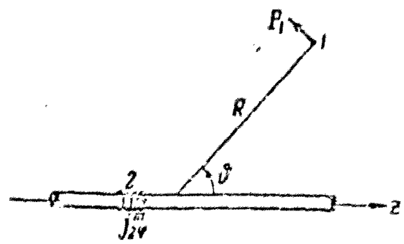

Figure 74.

The external current j_2^m is determined by the well-known equation

$$j_2^m = -\frac{c}{4\pi}[nE] \qquad (30.20)$$

in terms of the electric field **E** on the conductor's surface. In view of the boundary condition

$$E_z + E_z^e = 0 \qquad (30.21)$$

we have

$$j_{2\varphi}^m = -\frac{c}{4\pi} E_z^e. \qquad (30.22)$$

Furthermore, defining the dipole moment p_1 in terms of its field in free space (at the point x = y = z = 0)

$$E'_{0z} = -k^2 p_1 \frac{e^{ikR}}{R} \sin\vartheta \qquad (30.23)$$

and changing from the magnetic intensity $H_{1\phi}$ to the total current

$$J = \frac{ca}{2} H_{1\varphi}, \qquad (30.24)$$

induced by the dipole in the conductor, we obtain from the Lorentz lemma the following relationship:

$$E_{2\vartheta} = H_{2\varphi} = \frac{k^2 \sin\vartheta}{i\omega E'_{0z}} \cdot \frac{e^{ikR}}{R} \cdot \int_{-\infty}^{z_2} E_z^e J(z) dz. \qquad (30.25)$$

If the dipole p_1 is moved to a distance R >> $z_2 - z_0$, then the field radiated by it may be investigated on the section $z_2 - z_0$ of the semi-infinite conductor $(-\infty, z_2)$ as a plane wave. Then the current induced in this section of the conductor will be determined by the equation

$$J(z)=S'[e^{iwz}-e^{iwz_2}\psi_+(z_2-z)e^{ik(z_2-z)}], \tag{30.26}$$

where

$$S'=\frac{i\omega E_{0z}^{e}}{2k^2\sin^2\vartheta\ln\frac{2i}{\gamma ka\sin\vartheta}},\quad w=-k\cos\vartheta. \tag{30.27}$$

We will select the quantity z_0 in such a way that, at a distance $z_2 - z_0$ from the conductor end, the reflected current wave would be practically equal to zero $(\psi_+(z_2-z_0)\approx 0)$. Substituting the function (30.26) into the right-hand member of Equality (30.25) and taking for the quantity E_z^e the external field (30.16), we obtain

$$\begin{aligned} E_{2\vartheta}=H_{2\varphi}=&\frac{1}{2\sin\vartheta\ln\frac{2i}{\gamma ka\sin\vartheta}}\times\\ &\times\frac{e^{ikR}}{R}\int_{-\infty}^{z_2}E_z^e[e^{iwz}-\psi_+(z_2-z)e^{iwz_2+ik(z_2-z)}]dz=\\ =&\frac{1}{2\sin\vartheta\ln\frac{2i}{\gamma ka\sin\vartheta}}\frac{e^{ikR}}{R}\Big\{\mathcal{E}_1[e^{iwz_1}-\psi_+(L)e^{iwz_1+ikL}]+\\ &+\hat{E}_{0z}\frac{e^{i(w+w_0)z_1}}{i(w+w_0)}-\hat{E}_{0z}e^{i(k+w)z_2}\int_{-\infty}^{z_1}e^{-i(k-w_0)z}\psi_+(z_2-z)dz.\Big\} \end{aligned} \tag{30.28}$$

An important feature of this relationship is that the integration is performed here not along the entire conductor $(-\infty, z_2)$, but only along part of it $(-\infty, z_1)$, where the function $\psi_+(z_2-z)$ describes the current with good precision. The integrals here are calculated the same as in Equation (30.09). As a result, the field radiated by the semi-infinite conductor $(-\infty, z_2)$ will equal

$$E_{2\vartheta}=H_{2\varphi}=\frac{1}{2\sin\vartheta\ln\frac{2i}{\gamma ka\sin\vartheta}}\frac{e^{ikR}}{R}\times$$

(Equation continued on next page)

$$\times\left\{\mathcal{E}_1\left[e^{iwz_1}-\psi_+(L)\,e^{iwz_1+ikL}\right]-\frac{\hat{E}_{0z}e^{i(w+w_0)z_1}}{ik(\cos\vartheta+\cos\vartheta_0)}+\right.$$

$$\left.+\frac{\hat{E}_{0z}e^{ikL+i(w_0z_1+wz_2)}}{ik(\cos\vartheta+\cos\vartheta_0)}\left[\psi_+(L)-\frac{\sin^2\frac{\vartheta}{2}\ln\frac{i}{\gamma ka\sin\frac{\vartheta}{2}}}{\cos^2\frac{\vartheta_0}{2}\ln\frac{i}{\gamma ka\cos\frac{\vartheta_0}{2}}}\psi_-^0(L)\right]\right\}. \qquad (30.29)$$

The terms in the braces having the phase factor e^{iwz} correspond to the desired secondary wave diverging from the conductor's end $z = z_2$. Using Equations (30.13) and (30.15), this wave may be represented in the form

$$E_\vartheta^{(2)}(z_2)=H_\varphi^{(2)}(z_2)=\frac{\mathcal{E}^{(2)}(z_2)}{2\sin\vartheta\ln\frac{2i}{\gamma ka\sin\vartheta}}\frac{e^{ikR}}{R}e^{-ikz_2\cos\vartheta}, \qquad (30.30)$$

where

$$\mathcal{E}^{(2)}(z_2)=\frac{4iEe^{iw_0z_1+ikL}}{k\sin\vartheta_0(\cos\vartheta+\cos\vartheta_0)\ln\frac{2i}{\gamma ka\sin\vartheta_0}}\times$$

$$\times\left[\cos^2\frac{\vartheta_0}{2}\cos^2\frac{\vartheta}{2}\ln\frac{i}{\gamma ka\cos\frac{\vartheta_0}{2}}\psi_+(L)-\right.$$

$$\left.-\sin^2\frac{\vartheta_0}{2}\sin^2\frac{\vartheta}{2}\ln\frac{i}{\gamma ka\sin\frac{\vartheta}{2}}\psi_-^0(L)\right]. \qquad (30.31)$$

In a similar way, let us find the secondary diffraction wave being propagated from the end $z = z_1$. In order to do this, it is necessary to investigate the diffraction of primary wave (30.18) at the end $z = z_1$ of the semi-infinite conductor $(z_1\leqslant z\leqslant\infty)$. In this case, the principle of duality leads to the following relationship

$$E_{2\vartheta}=H_{2\varphi}=\frac{k^2\sin\vartheta}{i\omega E'_{0z}}\frac{e^{ikR}}{R}\int_{z_1}^{\infty}E_z^e\,J(z)\,dz, \qquad (30.32)$$

which, after substituting the function (30.17) and the current

$$J(z)=\frac{i\omega E'_{0z}}{2k^2\sin^2\vartheta\ln\frac{2i}{\gamma ka\sin\vartheta}}\left[e^{iwz}-\psi_-(z-z_1)\,e^{iwz_1+ik(z-z_1)}\right] \qquad (30.33)$$

in it gives us the field radiated by the semi-infinite conductor (z_1, ∞). The wave radiated by the conductor's end is the desired secondary edge wave and may be represented in the form

$$E_{\vartheta}^{(2)}(z_1)=H_{\varphi}^{(2)}(z_1)=\frac{\mathscr{E}^{(2)}(z_1)}{2\sin\vartheta\ln\frac{2i}{\gamma ka\sin\vartheta}}\frac{e^{ikR}}{R}e^{-ikz_1\cos\vartheta}, \qquad (30.34)$$

where

$$\begin{aligned}\mathscr{E}^{(2)}(z_1)=&-\frac{4iEe^{i\alpha_0 z_1+ikL}}{k\sin\vartheta_0(\cos\vartheta+\cos\vartheta_0)\ln\frac{2i}{\gamma ka\sin\vartheta_0}}\times\\&\times\left[\sin^2\frac{\vartheta_0}{2}\sin^2\frac{\vartheta}{2}\ln\frac{i}{\gamma ka\sin\frac{\vartheta_0}{2}}\psi_-(L)-\right.\\&\left.-\cos^2\frac{\vartheta_0}{2}\cos^2\frac{\vartheta}{2}\ln\frac{i}{\gamma ka\cos\frac{\vartheta}{2}}\psi_+^0(L)\right].\end{aligned} \qquad (30.35)$$

Otherwise, this expression may be written directly by replacing, in Equations (30.30) and (30.31), z_2 by z_1, ϑ by $\pi-\vartheta$ and ϑ_0 by $\pi-\vartheta_0$.

§ 31. Multiple Diffraction of Edge Waves

The secondary waves (30.30) and (30.34) which were found are the waves diverging from the ends of the semi-infinite conductors ($-\infty$, z_2) and (z_1, ∞). If one excites an infinite single-wire line by the external field

$$E_z^e=\mathscr{E}_2^{(2)}\delta(z-z_2), \qquad (31.01)$$

where

$$\mathscr{E}_2^{(2)}=\mathscr{E}^{(2)}(z_2)\Big|_{\vartheta=\pi}=E\frac{2i\ln\frac{i}{\gamma ka}\psi_-^{(0)}(L)}{k\sin\vartheta_0\ln\frac{2i}{\gamma ka\sin\vartheta_0}}e^{ikL-ikz_2\cos\vartheta_0}, \qquad (31.02)$$

then a spherical wave arises which with $\vartheta\approx\pi$ coincides with wave (30.30). With the excitation of an infinite line by the concentrated emf

$$E_z^e = \mathcal{E}_1^{(2)}\delta(z-z_1), \qquad (31.03)$$

where

$$\mathcal{E}_1^{(2)} = \mathcal{E}^{(2)}(z_1)\Big|_{\vartheta=0} = E\,\frac{2i\ln\frac{i}{\gamma ka}\,\psi_+^0(L)}{k\sin\vartheta_0\ln\frac{2i}{\gamma ka\sin\vartheta_0}}\,e^{ikL-ikz_2\cos\vartheta_0}, \qquad (31.04)$$

a wave arises which coincides with wave (30.34) when $\vartheta \approx 0$. It is not difficult to see that these external fields actually excite in an infinite single-wire line current waves which are equivalent to the secondary current waves in a passive vibrator [that is, equivalent to those waves which are expressed by the first terms in the brackets of Equation (28.18)]. Therefore, the tertiary waves may be investigated as edge waves radiated by the semi-infinite conductors (z_1, ∞) and ($-\infty$, z_2) with their excitation by the external fields (31.01) and (31.03), respectively. From Equations (30.25) and (30.32), we find without difficulty the total field radiated with the indicated excitation by the conductor (z_1, ∞)

$$E_\vartheta = H_\varphi = \frac{\mathcal{E}_2^{(2)}}{2\sin\vartheta\ln\frac{2i}{\gamma ka\sin\vartheta}}\,\frac{e^{ikR}}{R}\,[e^{iwz_1} - \psi_-(L)\,e^{iwz_1+ikL}] \qquad (31.05)$$

and the total field radiated by the conductor ($-\infty$, z_2)

$$E_\vartheta = H_\varphi = \frac{\mathcal{E}_1^{(2)}}{2\sin\vartheta\ln\frac{2i}{\gamma ka\sin\vartheta}}\,\frac{e^{ikR}}{R}\,[e^{iwz_1} - \psi_+(L)\,e^{iwz_2+ikL}]. \qquad (31.06)$$

As a result, we obtain for the tertiary waves diverging from the ends z_1 and z_2 the following expressions:

$$E_\vartheta^{(3)}(z_1) = H_\varphi^{(3)}(z_1) = \frac{\mathcal{E}^{(3)}(z_1)}{2\sin\vartheta\ln\frac{2i}{\gamma ka\sin\vartheta}}\,\frac{e^{ikR}}{R}\,e^{-ikz_1\cos\vartheta}, \qquad (31.07)$$

$$E_\vartheta^{(3)}(z_2) = H_\varphi^{(3)}(z_2) = \frac{\mathcal{E}^{(3)}(z_2)}{2\sin\vartheta\ln\frac{2i}{\gamma ka\sin\vartheta}}\,\frac{e^{ikR}}{R}\,e^{-ikz_2\cos\vartheta}, \qquad (31.08)$$

where

$$\left.\begin{aligned}\mathcal{E}^{(3)}(z_1) &= -\mathcal{E}_2^{(2)}\cdot\psi_-(L)\,e^{ikL},\\ \mathcal{E}^{(3)}(z_2) &= -\mathcal{E}_1^{(2)}\cdot\psi_+(L)\,e^{ikL}.\end{aligned}\right\} \tag{31.09}$$

In the directions toward the opposite end of the conductor, these waves are equivalent to the radiation of an infinite single-wire line excited by the external fields

$$E_z^e = \mathcal{E}_1^{(3)}\cdot\delta(z - z_1), \quad \mathcal{E}_1^{(3)} = \mathcal{E}^{(3)}(z_1)\Big|_{\vartheta=0}, \tag{31.10}$$

$$E_z^e = \mathcal{E}_2^{(3)}\cdot\delta(z_2 - z), \quad \mathcal{E}_2^{(3)} = \mathcal{E}^{(3)}(z_2)\Big|_{\vartheta=\pi}. \tag{31.11}$$

Consequently, the quaternary waves again may be investigated as edge waves radiated by the semi-infinite conductors ($-\infty$, z_2) and (z_1, ∞) with their excitation by the external fields (31.10) and (31.11). Using the reciprocity principle, we easily obtain

$$\left.\begin{aligned}E_\vartheta^{(4)}(z_1) = H_\varphi^{(4)}(z_1) &= \frac{\mathcal{E}^{(4)}(z_1)}{2\sin\vartheta\ln\dfrac{2i}{\gamma ka\sin\vartheta}}\,\frac{e^{ikR}}{R}\,e^{-ikz_1\cos\vartheta},\\ E_\vartheta^{(4)}(z_2) = H_\varphi^{(4)}(z_2) &= \frac{\mathcal{E}^{(4)}(z_2)}{2\sin\vartheta\ln\dfrac{2i}{\gamma ka\sin\vartheta}}\,\frac{e^{ikR}}{R}\,e^{-ikz_2\cos\vartheta},\end{aligned}\right\} \tag{31.12}$$

where

$$\left.\begin{aligned}\mathcal{E}^{(4)}(z_1) &= -\mathcal{E}_2^{(3)}\cdot\psi_-(L)\,e^{ikL},\\ \mathcal{E}^{(4)}(z_2) &= -\mathcal{E}_1^{(3)}\cdot\psi_+(L)\,e^{ikL},\end{aligned}\right\} \tag{31.13}$$

In a completely similar way, the n^{th} order edge waves

$$\left.\begin{aligned}E_\vartheta^{(n)}(z_1) = H_\varphi^{(n)}(z_1) &= \frac{\mathcal{E}^{(n)}(z_1)}{2\sin\vartheta\ln\dfrac{2i}{\gamma ka\sin\vartheta}}\,\frac{e^{ikR}}{R}\,e^{-ikz_1\cos\vartheta},\\ E_\vartheta^{(n)}(z_2) = H_\varphi^{(n)}(z_2) &= \frac{\mathcal{E}^{(n)}(z_2)}{2\sin\vartheta\ln\dfrac{2i}{\gamma ka\sin\vartheta}}\,\frac{e^{ikR}}{R}\,e^{-ikz_2\cos\vartheta}.\end{aligned}\right\} \tag{31.14}$$

are found. Here

$$\left.\begin{aligned}\mathcal{E}^{(n)}(z_1) &= -\mathcal{E}_2^{(n-1)}\cdot\psi_-(L)\,e^{ikL},\\ \mathcal{E}^{(n)}(z_2) &= -\mathcal{E}_1^{(n-1)}\cdot\psi_+(L)\,e^{ikL}\end{aligned}\right\} \tag{31.15}$$

and

$$\mathcal{E}_1^{(n)} = \mathcal{E}^{(n)}(z_1)\Big|_{\vartheta=0}, \quad \mathcal{E}_2^{(n)} = \mathcal{E}^{(n)}(z_2)\Big|_{\vartheta=\pi}. \tag{31.16}$$

Thus, the field arising with multiple diffractions (starting with the second) may be represented in the following form:

$$\begin{gathered}\sum_{n=2}^{\infty}[E_\vartheta^{(n)}(z_1) + E_\vartheta^{(n)}(z_2)] = \\ = \frac{1}{2\sin\vartheta\ln\frac{2i}{\gamma ka\sin\vartheta}}\frac{e^{ikR}}{R}\left[\sum_{n=2}^{\infty}\mathcal{E}^{(n)}(z_1)\,e^{-ikz_1\cos\vartheta} + \right. \\ \left. + \sum_{n=2}^{\infty}\mathcal{E}^{(n)}(z_2)\,e^{-ikz_2\cos\vartheta}\right],\end{gathered} \tag{31.17}$$

where

$$\begin{gathered}\sum_{n=2}^{\infty}\mathcal{E}^{(n)}(z_1) = \\ = \mathcal{E}^{(2)}(z_1) + [\mathcal{E}_1^{(2)}\cdot\psi(L)\,e^{ikL} - \mathcal{E}_2^{(2)}]\frac{\psi_-(L)}{D}\,e^{ikL},\end{gathered} \tag{31.18}$$

$$\begin{gathered}\sum_{n=2}^{\infty}\mathcal{E}^{(n)}(z_2) = \\ = \mathcal{E}^{(2)}(z_2) + [\mathcal{E}_2^{(2)}\cdot\psi(L)\,e^{ikL} - \mathcal{E}_1^{(2)}]\frac{\psi_+(L)}{D}\,e^{ikL},\end{gathered} \tag{31.19}$$

and the functions $\mathcal{E}^{(2)}(z_{1,2})$ and $\mathcal{E}_{1,2}^{(2)}$ are determined by Equations (30.31), (30.35), (31.02) and (31.04). We will not write out here the rather unwieldy final expression for this field, but we will proceed with a calculation of the total field scattered by a vibrator.

§ 32. Total Fringing Field

Before beginning the derivation of the expression for the scattering characteristic, let us make the following observation. The functions ϕ which enter into Equation (30.04) satisfy relationships (30.05), and may be found by factoring. However, our investigation of the successive waves arising with diffraction at the conductor's ends was approximate. Therefore, it makes no sense to use the precise Expression (30.04) for the primary field. We shall use the approximation expressions for the function Φ

$$\left.\begin{aligned} \Phi(-k\cos\vartheta,\ -k\cos\vartheta_0) &= \ln\frac{i}{\gamma ka\sin\frac{\vartheta}{2}\sin\frac{\vartheta_0}{2}}, \\ \Phi(k\cos\vartheta,\ k\cos\vartheta_0) &= \ln\frac{i}{\gamma ka\cos\frac{\vartheta}{2}\cos\frac{\vartheta_0}{2}}, \end{aligned}\right\} \tag{32.01}$$

which were obtained by the variational method and have a precision which is sufficient for our purpose (see [83]). More precisely speaking, we will use approximation Equations (32.01) in conjunction with the rigorous Expression (30.05), and we will set

$$\left.\begin{aligned} \frac{1}{\Phi(-k\cos\vartheta,\ -k\cos\vartheta_0)} &= \frac{\ln\dfrac{i}{\gamma ka\cos\frac{\vartheta}{2}\cos\frac{\vartheta_0}{2}}}{\ln\dfrac{2i}{\gamma ka\sin\vartheta}\ln\dfrac{2i}{\gamma ka\sin\vartheta_0}}, \\ \frac{1}{\Phi(k\cos\vartheta,\ k\cos\vartheta_0)} &= \frac{\ln\dfrac{i}{\gamma ka\sin\frac{\vartheta}{2}\sin\frac{\vartheta_0}{2}}}{\ln\dfrac{2i}{\gamma ka\sin\vartheta}\ln\dfrac{2i}{\gamma ka\sin\vartheta_0}}. \end{aligned}\right\} \tag{32.02}$$

Then the primary field will equal

$$\begin{aligned} E_\vartheta^{(1)} = H_\varphi^{(1)} = &-\frac{iE}{2(\cos\vartheta+\cos\vartheta_0)\ln\dfrac{2i}{\gamma ka\sin\vartheta}\ln\dfrac{2i}{\gamma ka\sin\vartheta_0}}\cdot\frac{e^{ikR}}{kR}\times \\ &\times\left[\operatorname{ctg}\frac{\vartheta_0}{2}\operatorname{ctg}\frac{\vartheta}{2}\ln\frac{i}{\gamma ka\cos\frac{\vartheta}{2}\cos\frac{\vartheta_0}{2}}e^{-ikz_1(\cos\vartheta+\cos\vartheta_0)}-\right. \\ &\left.-\operatorname{tg}\frac{\vartheta_0}{2}\operatorname{tg}\frac{\vartheta}{2}\ln\frac{i}{\gamma ka\sin\frac{\vartheta}{2}\sin\frac{\vartheta_0}{2}}e^{-ikz_2(\cos\vartheta+\cos\vartheta_0)}\right]. \end{aligned} \tag{32.03}$$

Now summing Expressions (31.17) and (32.03), we find the total field scattered by a passive vibrator in the form

$$E_\vartheta = H_\varphi = -E \cdot \frac{e^{ikR}}{kR} \cdot F(\vartheta, \vartheta_0), \tag{32.04}$$

where

$$
\begin{aligned}
F(\vartheta, \vartheta_0) = {} & \frac{2i}{(\cos\vartheta + \cos\vartheta_0)\sin\vartheta\sin\vartheta_0 \ln\frac{2i}{\gamma ka\sin\vartheta}\ln\frac{2i}{\gamma ka\sin\vartheta_0}} \times \\
& \times \left\{ \cos^2\frac{\vartheta}{2}\cos^2\frac{\vartheta_0}{2}\ln\frac{i}{\gamma ka\cos\frac{\vartheta}{2}\cos\frac{\vartheta_0}{2}} e^{-ikz_1(\cos\vartheta + \cos\vartheta_0)} - \right. \\
& - \sin^2\frac{\vartheta}{2}\sin^2\frac{\vartheta_0}{2}\ln\frac{i}{\gamma ka\sin\frac{\vartheta}{2}\sin\frac{\vartheta_0}{2}} e^{-ikz_2(\cos\vartheta + \cos\vartheta_0)} + \\
& + e^{ikL}\left[\sin^2\frac{\vartheta}{2}\sin^2\frac{\vartheta_0}{2}\ln\frac{i}{\gamma ka\sin\frac{\vartheta_0}{2}}\psi_- - \right. \\
& \left. - \cos^2\frac{\vartheta}{2}\cos^2\frac{\vartheta_0}{2}\ln\frac{i}{\gamma ka\cos\frac{\vartheta}{2}}\psi_+^0\right] e^{-ik(z_2\cos\vartheta_0 + z_1\cos\vartheta)} + \\
& + e^{ikL}\left[\sin^2\frac{\vartheta}{2}\sin^2\frac{\vartheta_0}{2}\ln\frac{i}{\gamma ka\sin\frac{\vartheta}{2}}\psi_-^0 - \right. \\
& \left. - \cos^2\frac{\vartheta}{2}\cos^2\frac{\vartheta_0}{2}\ln\frac{i}{\gamma ka\cos\frac{\vartheta_0}{2}}\psi_+\right] e^{-ik(z_2\cos\vartheta + z_1\cos\vartheta_0)} - \\
& - \frac{(\cos\vartheta + \cos\vartheta_0)\ln\frac{i}{\gamma ka}}{2D} e^{2ikL}\left[\psi_+^0\psi e^{ikL - ikz_2\cos\vartheta_0} - \right. \\
& \left. - \psi_-^0 e^{-ikz_1\cos\vartheta_0}\right]\psi_- e^{-ikz_1\cos\vartheta} - \\
& - \frac{(\cos\vartheta + \cos\vartheta_0)\ln\frac{i}{\gamma ka}}{2D} e^{2ikL}\left[\psi_-^0\psi e^{ikL - ikz_1\cos\vartheta_0} - \right. \\
& \left. \left. - \psi_+^0 e^{-ikz_2\cos\vartheta_0}\right]\psi_+ e^{-ikz_2\cos\vartheta}\right\},
\end{aligned}
\tag{32.05}
$$

in which all the functions $\psi_\pm$ and $\psi_\pm^0$ have the argument L. The resulting expression, despite its complexity, has a clear physical meaning. Actually, the first term in the braces corresponds to the primary edge wave radiated by the conductor's end $z = z_1$; the second term corresponds to the primary wave radiated by the conductor's end $z = z_2$. The terms included in the first pair of brackets refer to the secondary wave departing from the end $z = z_1$, and the terms in the second set of brackets refer to the secondary wave departing from the end $z = z_2$. The remaining terms describe the sum of all subsequent waves arising with multiple diffraction and have a resonance character. The resonance begins with $L = z_2 - z_1 \approx n \cdot \lambda/2$ $(n = 1, 2, 3 \ldots)$ when $D \approx 0$.

Another important feature of the scattering characteristic is that it satisfies the reciprocity principle — that is, it does not change its value with a mutual interchange of ϑ and ϑ_0. One may also show that the vibrator does not radiate in the directions along its axis, and it does not scatter electromagnetic waves with glancing irradiation, that is,

$$F(0,\ \vartheta_0)=F(\pi,\ \vartheta_0)=F(\vartheta,\ 0)=F(\vartheta,\ \pi)=0. \tag{32.06}$$

Furthermore, using representation (28.25) for the functions ψ and $\psi_{\pm}$, we obtain the following expression for scattering characteristic (32.05) in the direction of the mirror-reflected ray $(\vartheta=\pi-\vartheta_0)$:

$$\begin{aligned} F(\pi-\vartheta_0,\ \vartheta_0)={}&-\frac{kL}{2\ln\dfrac{2i}{\gamma ka\sin\vartheta_0}}+\\ &+kL\,\frac{(\psi_{+}^{0})^2E\left(2kL\sin^2\dfrac{\vartheta_0}{2}\right)+(\psi_{-}^{0})^2E\left(2kL\cos^2\dfrac{\vartheta_0}{2}\right)}{4\left(\ln\dfrac{2i}{\gamma ka\sin\vartheta_0}\right)^2}+\\ &+\frac{i}{\left(\sin\vartheta_0\ln\dfrac{2i}{\gamma ka\sin\vartheta_0}\right)^2}\left\{\ln\frac{2i}{\gamma ka\sin\vartheta_0}-\frac{1}{2}+\right.\\ &+\left(\frac{1}{2}\psi_{+}^{0}\cos^2\frac{\vartheta_0}{2}-\ln\frac{i}{\gamma ka\sin\dfrac{\vartheta_0}{2}}\right)\psi_{+}^{0}e^{ikL(1-\cos\vartheta_0)}+\\ &+\left(\frac{1}{2}\psi_{-}^{0}\sin^2\frac{\vartheta_0}{2}-\ln\frac{i}{\gamma ka\cos\dfrac{\vartheta_0}{2}}\right)\psi_{-}^{0}\cdot e^{ikL(1+\cos\vartheta_0)}-\\ &-\frac{\ln\dfrac{i}{\gamma ka}}{D}e^{2ikL}\left[\psi_{+}^{0}\psi e^{ikL(1-\cos\vartheta_0)}-\psi_{-}^{0}\right]\psi_{+}^{0}-\\ &\left.-\frac{\ln\dfrac{i}{\gamma ka}}{D}e^{2ikL}\left[\psi_{-}^{0}\psi e^{ikL(1+\cos\vartheta_0)}-\psi_{+}^{0}\right]\psi_{-}^{0}\right\}. \end{aligned} \tag{32.07}$$

With glancing irradiation of a vibrator, when $\vartheta_0=0$ or $\vartheta_0=\pi$, it follows from this that $F(\pi,\ 0)=F(0,\ \pi)=0$.

Now assuming that $\vartheta_0=\frac{\pi}{2}$ in Equation (32.07), we obtain the relationship

$$\begin{aligned} F\left(\frac{\pi}{2},\ \frac{\pi}{2}\right)\dot{=}{}&-\frac{kL}{2\ln\dfrac{2i}{\gamma ka}}+kL\,\frac{\bar{\psi}^2\cdot E(kL)}{2\left(\ln\dfrac{2i}{\gamma ka}\right)^2}+\\ &+\frac{i}{\left(\ln\dfrac{2i}{\gamma ka}\right)^2}\left[\ln\frac{2i}{\gamma ka}-\frac{1}{2}+\left(\frac{1}{2}\bar{\psi}-2\ln\frac{\sqrt{2}\,i}{\gamma ka}\right)\bar{\psi}e^{ikL}+\right. \end{aligned}$$

Equation continued on next page

$$+2\bar{\psi}^2 \frac{\ln\frac{i}{\gamma ka}}{1+\bar{\psi}e^{ikL}} e^{2ikL}\Bigg],$$

$$\bar{\psi}=\psi^0_{\pm}\Big|_{\vartheta_0=\frac{\pi}{2}}, \tag{32.08}$$

which characterizes the reflected field magnitude with normal irradiation.

Let us also write the expressions for the function $F(\vartheta, \vartheta_0)$ which corresponds to the radar case when the observation and irradiation directions coincide $(\vartheta=\vartheta_0)$

$$F(\vartheta, \vartheta)=$$

$$=\frac{i}{\left(\sin\vartheta\ln\frac{2i}{\gamma ka\sin\vartheta}\right)^2\cos\vartheta}\Bigg\{\cos^4\frac{\vartheta}{2}\ln\frac{i}{\gamma ka\cos^2\frac{\vartheta}{2}}e^{-2ikz_1\cos\vartheta}-$$

$$-\sin^4\frac{\vartheta}{2}\ln\frac{i}{\gamma ka\sin^2\frac{\vartheta}{2}}e^{-2ikz_2\cos\vartheta}+$$

$$+2\Bigg[\sin^4\frac{\vartheta}{2}\ln\frac{i}{\gamma ka\sin\frac{\vartheta}{2}}\psi_-(L)-$$

$$-\cos^4\frac{\vartheta}{2}\ln\frac{i}{\gamma ka\cos\frac{\vartheta}{2}}\psi_+(L)\Bigg]e^{-ik(z_1+z_2)\cos\vartheta+ikL}-$$

$$-2\frac{\cos\vartheta}{D}\ln\frac{i}{\gamma ka}\psi(L)\psi_+(L)\psi_-(L)e^{-ik(z_1+z_2)\cos\vartheta+3ikL}+$$

$$+\frac{\cos\vartheta}{D}\ln\frac{i}{\gamma ka}\cdot[\psi_-^2(L)e^{-2ikz_1\cos\vartheta}+$$

$$+\psi_+^2(L)e^{-2ikz_2\cos\vartheta}]e^{2ikL}\Bigg\}. \tag{32.09}$$

We may show that when $\vartheta=\frac{\pi}{2}$ Equation (32.09) leads to Expression (32.08).

The scattering characteristic (32.05) which was found above was obtained by summing all the waves formed with multiple diffraction. Such a method is very graphic, but somewhat lengthy. One may arrive at the same result more quickly if it is assumed that the edge wave diffraction process at the passive vibrator's ends takes place, starting with tertiary action, the same as at the end of a transmitting vibrator. Therefore, the passive vibrator's scattering diagram may be directly sought in the form

$$F(\vartheta, \vartheta_0) = \frac{2i}{\sin\vartheta \sin\vartheta_0 \ln\frac{2i}{\gamma ka \sin\vartheta} \ln\frac{2i}{\gamma ka \sin\vartheta_0}} \cdot f(\vartheta, \vartheta_0), \tag{32.10}$$

where

$$\begin{aligned} f(\vartheta, \vartheta_0) = &\frac{1}{\cos\vartheta + \cos\vartheta_0} \times \\ \times \Bigg\{ &\cos^2\frac{\vartheta}{2}\cos^2\frac{\vartheta_0}{2} \ln\frac{i}{\gamma ka \cos\frac{\vartheta}{2}\cos\frac{\vartheta_0}{2}} e^{-ikz_1(\cos\vartheta + \cos\vartheta_0)} - \\ &- \sin^2\frac{\vartheta}{2}\sin^2\frac{\vartheta_0}{2} \ln\frac{i}{\gamma ka \sin\frac{\vartheta}{2}\sin\frac{\vartheta_0}{2}} e^{-ikz_2(\cos\vartheta + \cos\vartheta_0)} + \\ &+ e^{ikL}\Big[\sin^2\frac{\vartheta}{2}\sin^2\frac{\vartheta_0}{2} \ln\frac{i}{\gamma ka \sin\frac{\vartheta_0}{2}} \psi_-(L) - \\ &- \cos^2\frac{\vartheta}{2}\cos^2\frac{\vartheta_0}{2} \ln\frac{i}{\gamma ka \cos\frac{\vartheta}{2}} \psi_+^*(L)\Big] e^{-ik(z_2\cos\vartheta_0 + z_1\cos\vartheta)} + \\ &+ e^{ikL}\cdot\Big[\sin^2\frac{\vartheta}{2}\sin^2\frac{\vartheta_0}{2} \ln\frac{i}{\gamma ka \sin\frac{\vartheta}{2}} \psi_-^*(L) - \\ &- \cos^2\frac{\vartheta}{2}\cos^2\frac{\vartheta_0}{2} \ln\frac{i}{\gamma ka \cos\frac{\vartheta_0}{2}} \psi_+(L)\Big] e^{-ik(z_2\cos\vartheta + z_1\cos\vartheta_0)}\Bigg\} + \\ &+ C_1\psi_-(L) e^{-ikz_1\cos\vartheta} + C_2\psi_+(L) e^{-ikz_2\cos\vartheta}. \end{aligned} \tag{32.11}$$

The last two terms in Equation (32.11) are the sum of all edge waves starting with the tertiary waves which are propagated from the conductor's ends $z = z_1$ and $z = z_2$, respectively. The constants C_1 and C_2 are determined from the condition

$$f(0, \vartheta_0) = f(\pi, \vartheta_0) = 0, \tag{32.12}$$

which leads to the system of equations

$$\left.\begin{aligned} C_1\psi(L) e^{ikL} + C_2 &= \frac{1}{2}\ln\frac{i}{\gamma ka}\psi_+^*(L) e^{2ikL - ikz_2\cos\vartheta_0}, \\ C_1 + C_2\psi(L) e^{ikL} &= \frac{1}{2}\ln\frac{i}{\gamma ka}\psi_-^*(L) e^{2ikL - ikz_1\cos\vartheta_0}. \end{aligned}\right\} \tag{32.13}$$

from which

$$
\left.
\begin{aligned}
C_1 &= -\frac{1}{2D} e^{2ikL} \cdot \ln \frac{i}{\gamma ka} \cdot [\psi_+^{\bullet} \psi e^{ikL - ikz_2 \cos\vartheta_0} - \\
&\qquad - \psi_-^{0} e^{-ikz_1 \cos\vartheta_0}], \\
C_2 &= -\frac{1}{2D} e^{2ikL} \cdot \ln \frac{i}{\gamma ka} \cdot [\psi_-^{\bullet} \psi e^{ikL - ikz_1 \cos\vartheta_0} - \\
&\qquad - \psi_+^{\bullet} e^{-ikz_2 \cos\vartheta_0}].
\end{aligned}
\right\} \tag{32.14}
$$

§ 33. A Vibrator Which is Short in Comparison With the Wavelength (a Passive Dipole)

The theory of plane wave scattering by a thin cylindrical vibrator which is discussed in this chapter is based on a number of physical considerations. One good aspect of this theory is the fact that its precision increases with the length of the vibrator, since the current waves whose diffraction we are investigating are expressed more clearly, the longer the vibrator. However, one may also show that for short vibrators, the length of which is small in comparison with the wavelength, the equations derived by us have good precision.

It is clear that a vibrator which is short in comparison with the wavelength acts as a dipole, creating a fringing field

$$E = H_\varphi = -k^2 p_z \frac{e^{ikR}}{kR} \sin\vartheta, \tag{33.01}$$

where the dipole moment p_z may be calculated by solving the electrostatic problem. This dipole moment depends on the dimensions and shape of the vibrator. In accordance with [92], the dipole moment of a cylinder in a uniform electrostatic field E_z equals

$$p_z = D(l)\left(\frac{L}{2}\right)^3 \cdot E_z, \tag{33.02}$$

where $D(l)$ is a dimensionless function $l = L/2a$ which is shown in Figure 75 by the continuous curve. With $l \gg 1$, one may calculate the function $D(l)$ by means of the asymptotic expansion

$$D(l) = \frac{2}{3}\left(\frac{1}{\Omega_2} + \frac{0,977}{\Omega_2^3} + \ldots\right), \quad \Omega_2 = 2\left(\ln 4l - \frac{7}{3}\right). \tag{33.03}$$

If in this expansion one limits oneself to the first term, then

$$D(l) = \frac{1}{3\left(\ln 4l - \frac{7}{3}\right)}. \tag{33.04}$$

The results of numerical calculations based on Equations (33.04) are shown in Figure 75 by the dashed curve: we see that the latter equation gives a good precision already with $l \gtrsim 9$.

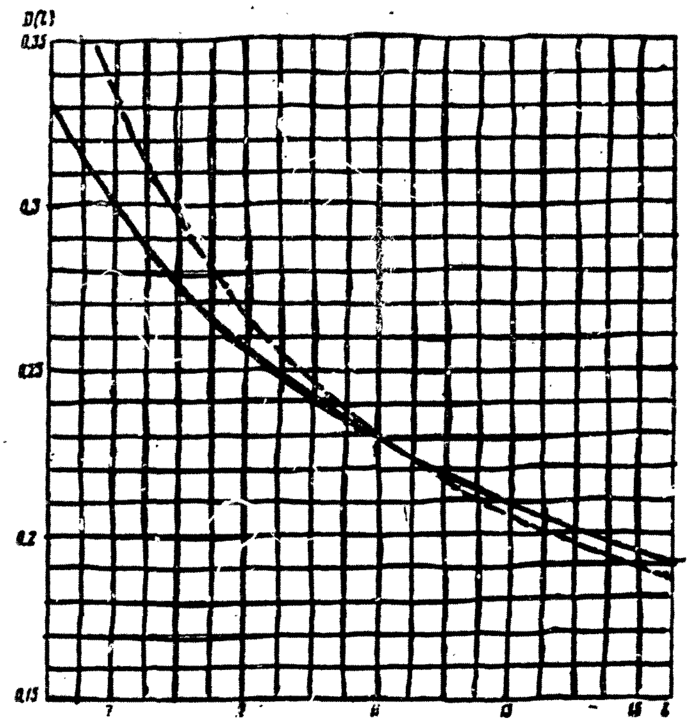

Figure 75. Graph of the function $D(l)$ which determines the cylinder's dipole moment.

Thus, the dipole moment of a vibrator which is short in comparison with the wavelength equals

$$p_z = E\frac{L^3}{24}\frac{\sin\vartheta_0}{\ln\frac{2L}{a} - \frac{7}{3}}\left\{1 + O\left[\left(\ln\frac{L}{a}\right)^{-2}\right]\right\}, \tag{33.05}$$

and its scattering characteristic must have the form

$$F(\vartheta, \vartheta_0) = \frac{k^2L^2}{24} \frac{\sin\vartheta \sin\vartheta_0}{\ln\frac{2L}{a} - \frac{7}{3}} \left\{1 + O\left[\left(\ln\frac{L}{a}\right)^{-2}\right]\right\}. \tag{33.06}$$

In this section we find the first two terms of the expansion of the vibrator's scattering characteristic F in reciprocal powers of the large parameter L/a (with $\lambda \to \infty$), and we compare them with Expression (33.06). We shall limit ourselves to the case $\vartheta = \vartheta_0 = \frac{\pi}{2}$, when the function F is described by the simpler Equation (32.08).

With small values of the argument z, the functions $\psi(z)$ and $\bar{\psi}(z) = \psi_+|_{\vartheta=\frac{\pi}{2}} = \psi_-|_{\vartheta=\frac{\pi}{2}}$ [see Equations (28.21)] may be represented in the form

$$\left.\begin{aligned} \psi(z) &= 1 - \frac{g(z) - g(0)}{2g(0)} + O\left(\frac{1}{g^2(0)}\right), \\ \bar{\psi}(z) &= 1 - \frac{\bar{g}(z) - \bar{g}(0)}{2g(0)} + O\left(\frac{1}{g^2(0)}\right). \end{aligned}\right\} \tag{33.07}$$

The functions g and $\bar{g}$ included here are determined by the equations:

$$g(z) - g(0) = \ln\frac{2\gamma kz}{i} + e^{-2ikz}\int_z^\infty \frac{e^{2ik\sigma}}{\sigma}\,d\sigma,\ g(0) = \ln\frac{i}{\gamma ka} \tag{33.08}$$

and

$$\bar{g}(z) - \bar{g}(0) = \ln\frac{\gamma kz}{i} + e^{-ikz}\int_z^\infty \frac{e^{ik\sigma}}{\sigma}\,d\sigma,\ \bar{g}(0) = \ln\frac{2i}{\gamma ka}. \tag{33.09}$$

Let us note that Expressions (33.07) completely agree with the corresponding terms of the aysmptotic expansion for the functions ψ and $\bar{\psi}$, which may be obtained from the initial integral equations which determine these functions (see, for example, [81], § 4).

Limiting ourselves in the expansion for the functions $\psi(z)$ and $\bar{\psi}(z)$ to terms of the order of $(kz)^3$, we have

$$\psi(z)=1-\frac{1}{\ln\frac{i}{\gamma ka}}\left[ikz\left(\ln\frac{2\gamma kz}{i}-1\right)+\right.$$
$$\left.+k^2z^2\left(\ln\frac{2\gamma kz}{i}-\frac{3}{2}\right)-i\frac{2}{3}k^3z^3\left(\ln\frac{2\gamma kz}{i}-\frac{11}{6}\right)\right] \tag{33.10}$$

and

$$\bar{\psi}(z)=1-\frac{1}{2\ln\frac{i}{\gamma ka}}\left[ikz\left(\ln\frac{\gamma kz}{i}-1\right)+\right.$$
$$\left.+\frac{k^2z^2}{2}\left(\ln\frac{\gamma kz}{i}-\frac{3}{2}\right)-i\frac{k^3z^3}{6}\left(\ln\frac{\gamma kz}{i}-\frac{11}{6}\right)\right]. \tag{33.11}$$

In addition, terms of the order $\left(\ln\frac{i}{\gamma ka}\right)^{-2}$ are omitted in Expressions (33.10) and (33.11). Now if we substitute these expressions into Equation (32.08), then one should omit terms of the order $\left(\ln\frac{i}{\gamma ka}\right)^{-3}$ in it. Therefore, the function $F\left(\frac{\pi}{2},\frac{\pi}{2}\right)$ may be represented in the form

$$F\left(\frac{\pi}{2},\frac{\pi}{2}\right)=\frac{i}{\left(\ln\frac{2i}{\gamma ka}\right)^2}\left\{\frac{ikL}{2}\left[\ln\frac{2i}{\gamma ka}-E(kL)\right]+\right.$$
$$+\ln\frac{2i}{\gamma ka}-\frac{1}{2}+\frac{1}{2}e^{ikL}-2\bar{\psi}(L)\ln\frac{\sqrt{2}\,i}{\gamma ka}e^{ikL}+$$
$$\left.+\frac{2\ln\frac{i}{\gamma ka}}{1+\psi(L)e^{ikL}}\bar{\psi}^2(L)e^{2ikL}\right\}. \tag{33.12}$$

Furthermore, taking into account Equations (33.10) and (33.11), we find

$$\frac{ikL}{2}\left[\ln\frac{2i}{\gamma ka}-E(kL)\right]=$$
$$=\frac{ikL}{2}\left(\ln\frac{2i}{\gamma ka}-\ln\frac{\gamma kL}{i}-ikL+\frac{k^2L^2}{4}\right), \tag{33.13}$$

$$\ln\frac{2i}{\gamma ka}-\frac{1}{2}+\frac{1}{2}e^{ikL}=$$
$$=\ln\frac{2i}{\gamma ka}+\frac{ikL}{2}-\frac{k^2L^2}{4}-i\frac{k^3L^3}{12}, \tag{33.14}$$

$$2\bar{\psi}(L)\ln\frac{\sqrt{2}\,i}{\gamma ka}e^{ikL}=2\left(1+ikL-\frac{k^2L^2}{2}-i\frac{k^3L^3}{6}\right)\ln\frac{\sqrt{2}\,i}{\gamma ka}-$$
$$-ikL\left(\ln\frac{\gamma kL}{i}-1\right)+k^2L^2\left(\frac{1}{2}\ln\frac{\gamma kL}{i}-\frac{1}{4}\right)+$$
$$+ik^3L^3\left(\frac{1}{6}\ln\frac{\gamma kL}{i}-\frac{1}{18}\right), \tag{33.15}$$

and finally

$$\frac{2\ln\frac{i}{\gamma ka}}{1+\psi(L)e^{ikL}}\,\bar{\psi}^2(L)\,e^{2ikL}=$$
$$=\left(1+i\frac{3}{2}kL-k^2L^2-i\frac{9}{24}k^3L^3\right)\ln\frac{i}{\gamma ka}-$$
$$-\frac{ikL}{2}\ln\frac{\gamma kL}{i}+\frac{ikL}{2}(1+\ln 2)+\frac{k^2L^2}{2}\ln\frac{\gamma kL}{i}-\frac{k^2L^2}{2}(1+\ln 2)+$$
$$+i\frac{5}{24}k^3L^3\ln\frac{\gamma kL}{i}-ik^3L^3\left(\frac{14}{72}+\frac{5\ln 2}{24}\right). \tag{33.16}$$

Using these relationships, it is not difficult to show that

$$F\left(\frac{\pi}{2},\ \frac{\pi}{2}\right)=\frac{k^3L^3}{24}\left\{\frac{1}{\ln\frac{L}{a}}+\frac{\frac{7}{3}-\ln 2}{\left(\ln\frac{L}{a}\right)^2}+O\left[\left(\ln\frac{L}{a}\right)^{-3}\right]\right\}. \tag{33.17}$$

The equation which has been found may be rewritten in the form

$$F\left(\frac{\pi}{2};\ \frac{\pi}{2}\right)=\frac{k^3L^3}{24\left(\ln\frac{2L}{a}-\frac{7}{3}\right)}\left\{1+O\left[\left(\ln\frac{L}{a}\right)^{-2}\right]\right\}. \tag{33.18}$$

It completely agrees with Equation (33.06) which follows from [92].

This result confirms the correctness of scattering characteristic (32.05) calculated by us, and shows that it is applicable for vibrators of any length.

§ 34. The Results of Numerical Calculations

The function $F(\vartheta, \vartheta_0)$ enables one to calculate the integral scattering thickness S and the effective scattering area σ of a passive vibrator. The integral scattering thickness is determined by the relationship

$$S=\frac{P}{p}, \tag{34.01}$$

where

$$p=\frac{c}{8\pi^2}E_0^2 \tag{34.02}$$

is the energy flux density in the incident wave averaged over an oscillation period, and

$$P=\frac{c}{8\pi}\operatorname{Re}\int \mathrm{n}[\mathrm{E}\mathrm{H}^*]dS=\frac{1}{2}E\sin\vartheta_0\operatorname{Re}\int_{z_1}^{z_2}J(z)\,e^{ikz\cos\vartheta_0}dz \tag{34.03}$$

is the value of the energy scattered by the vibrator into the surrounding space averaged over a period. Since one may represent the fringing field in the far zone in the direction $\vartheta=\pi-\vartheta_0$ by the equations

$$E_\vartheta=H_\varphi=-\frac{ik}{c}\sin\vartheta_0\frac{e^{ikR}}{R}\cdot\int_{z_1}^{z_2}J(z)\,e^{ikz\cos\vartheta_0}dz \tag{34.04}$$

and

$$E_\vartheta=H_\varphi=-E\frac{e^{ikR}}{kR}\cdot F(\pi-\vartheta_0,\vartheta_0), \tag{34.05}$$

then, having determined from this the integral

$$\int_{z_1}^{z_2}J(z)\,e^{ikz\cos\vartheta_0}dz=\frac{cE}{ik^2\sin\vartheta_0}F(\pi-\vartheta_0,\vartheta_0) \tag{34.06}$$

we obtain

$$S=\frac{\lambda^2}{\pi}\cos^2\alpha\cdot\operatorname{Im}F(\pi-\vartheta_0,\vartheta_0). \tag{34.07}$$

Calculations of the quantity S/L^2 (with $\alpha=0,\ \vartheta_0=\frac{\pi}{2}$), performed by us for vibrators with the parameter $\chi=\frac{1}{2\ln ka}$ taking the values $\chi = -0.05$ and $\chi = -0.1$, are found to be in agreement with the results of Leontovich and Levin [85]. With $\chi = -0.1$, our curve (the dotted line in Figure 76) is only slightly displaced in the direction of longer wavelengths and gives slightly higher resonance peaks.

The effective scattering area σ according to the definition equals

$$\sigma(\vartheta,\vartheta_0)=\frac{P_r\cdot 4\pi R^2}{p}, \tag{34.08}$$

where p is the known quantity (34.02) and

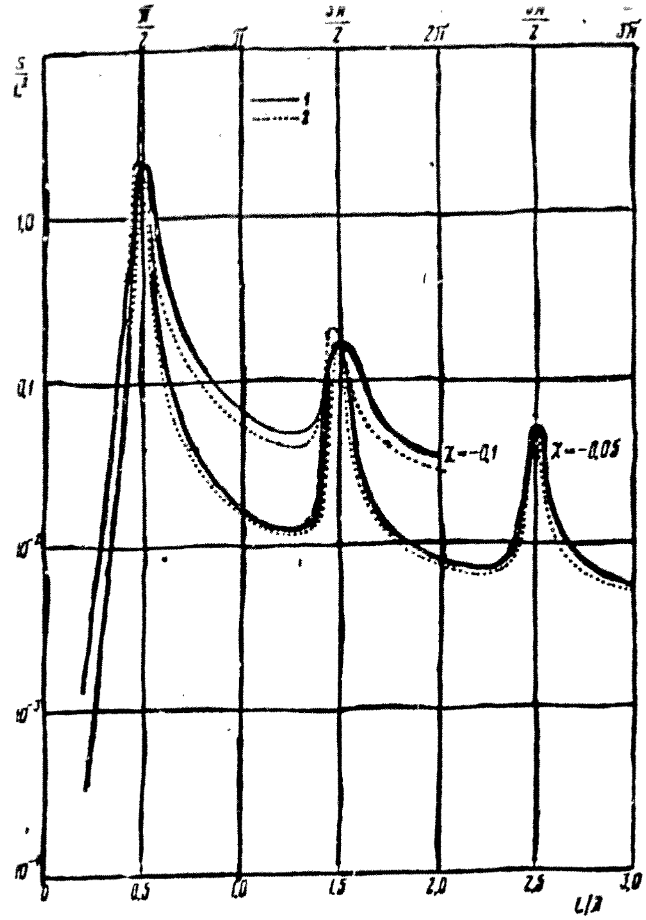

Figure 76. The integral scattering thickness of a vibrator as a function of its length (with normal incidence of a plane wave). Curves 1 were calculated by Leontovich and Levin [85]. Curves 2 were calculated on the basis of Equation (34.07).

$$P_r = \frac{c}{8\pi}|E_\vartheta|^2 = \frac{c}{8\pi}\frac{|F(\vartheta, \vartheta_0)|^2}{k^2R^2}E^2 \tag{34.09}$$

represents the average value of the energy flux density scattered by the vibrator in the direction ϑ. Consequently

$$\sigma(\vartheta, \vartheta_0) = \frac{\lambda^2}{\pi}\cos^2\alpha \cdot |F(\vartheta, \vartheta_0)|^2. \tag{34.10}$$

If the receiving antenna operates with the same polarization as the transmitting antenna, then the corresponding value of the effective area will equal

$$\sigma_{\alpha}(\vartheta, \vartheta_0) = \frac{\lambda^2}{\pi} \cos^4\alpha \cdot |F(\vartheta, \vartheta_0)|^2 \tag{34.11}$$

In the case of radar when the transmitting and receiving antennas are combined and the polarization is arbitrary, the vibrator's scattering properties are characterized by the average value

$$\bar{\sigma}(\vartheta) = \frac{1}{2\pi}\int_0^{2\pi} \sigma_{\alpha}(\vartheta, \vartheta_0)\, d\alpha = \frac{3\lambda^2}{8\pi} |F(\vartheta, \vartheta_0)|^2. \tag{34.12}$$

In Figures 77 and 78, the results of calculations performed on the basis of Equations (34.12) and (32.08) for the case of normal irradiation $\left(\vartheta = \frac{\pi}{2}\right)$ are represented by the dotted lines. Figure 77 illustrates the dependence of the function $\bar{\sigma}$ on the quantity kL with a given value of $\Omega_p = 2\ln\frac{2L}{a} = 15$, — that is, when the ratio of the vibrator's length to its diameter equals $L/2a = 452$. In Figure 78 the graph of the function $\bar{\sigma}$ is constructed as a function of the frequency $f = c/\lambda \cdot 10^{-6}$ (in megahertz) for the prescribed parameters $L = 5$ cm and $\Omega_p = 15$. Here the curves plotted by Lindroth [79] are drawn with a continuous line, and the curve in Figure 77 calculated by Van Vleck et al. [86] is traced by the dash-dot-curve.

The curves of Lindroth and Van Vleck were calculated by integrating the current which is found as a result of the approximate solution of the integral equation. However, this procedure was performed in [79] and [86] in a different way. Lindroth obtained an expression for the fringing field in the form of an expansion in reciprocal powers of the parameter Ω_p. The expression includes terms of the order of Ω_p^{-3}. In [86] a different kind of approximation was used which led, as can be seen from Figure 77, to rather rough results especially in the resonance region. Our curve (the dotted area) agrees almost everywhere within the limits of graphical precision with the curve of Lindroth. A noticeable divergence is observed only in the magnitude of the first resonance peak.

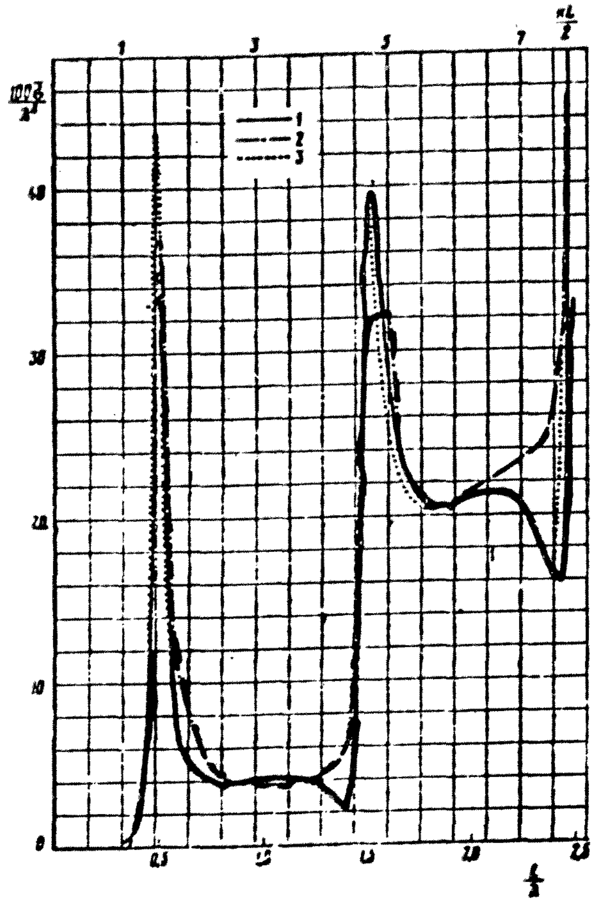

Figure 77. The effective scattering area of a vibrator as a function of its length with normal incidence of a plane wave. Curve 1 was calculated by Lindroth [79]; curve 2 was calculated by Van Vleck [86] by means of the method of integral equations; curve 3 was calculated on the basis of Equation (34.12).

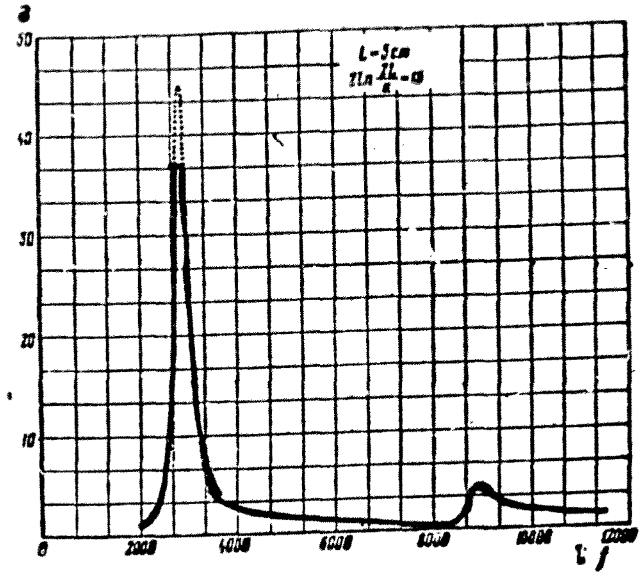

Figure 78. The effective scattering area of a vibrator as a function of the frequency $f = c/\lambda \cdot 10^{-6}$ (in megahertz) with normal incidence of a plane wave. The designations are the same as those in Figure 77.

In Figure 79 and 80 radar diagrams are constructed for vibrators of a length $L = 0.5\lambda$ and $L = 2\lambda$ with the specified value $L/a = 900$. Curves 1 were calculated by Tai using the variational method [87]. Curves 3 were obtained by the method of induced emf [86]; curves 4 were obtained by the above-indicated method of Van Vleck. The results of calculations based on our Equations (34.12), (32.08) and (32.09) are shown by curves 2.

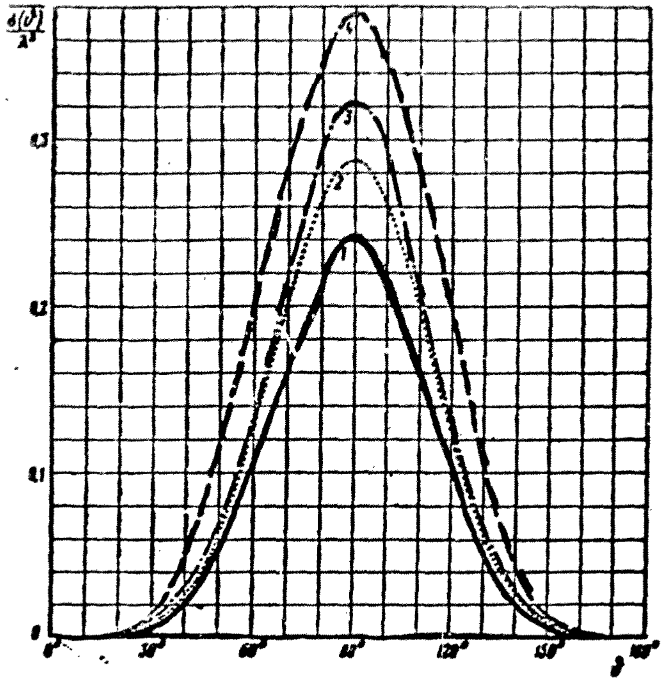

Figure 79. A comparison of the diagrams for the effective scattering area of a half-wave vibrator calculated by various methods.

Curve 1 was calculated by Tai [87] by the variational method;

Curve 2 was calculated on the basis of Equation (34.12);

Curve 3 was calculated by the method of induced emf (in the work of Van Vleck [86]);

Curve 4 was calculated by Van Vleck [86] by the method of integral equations.

In the cited references, the fringing field was calculated by the direct integration of the current. In order to determine the current, various approximation methods were used. In the variational method [87] a functional was constructed for this purpose which was stationary in respect to small current variations. Then the current was sought in the form of some function containing undetermined constants. These constants were found from the condition of the functional's stationarity. This method enables one to rather easily

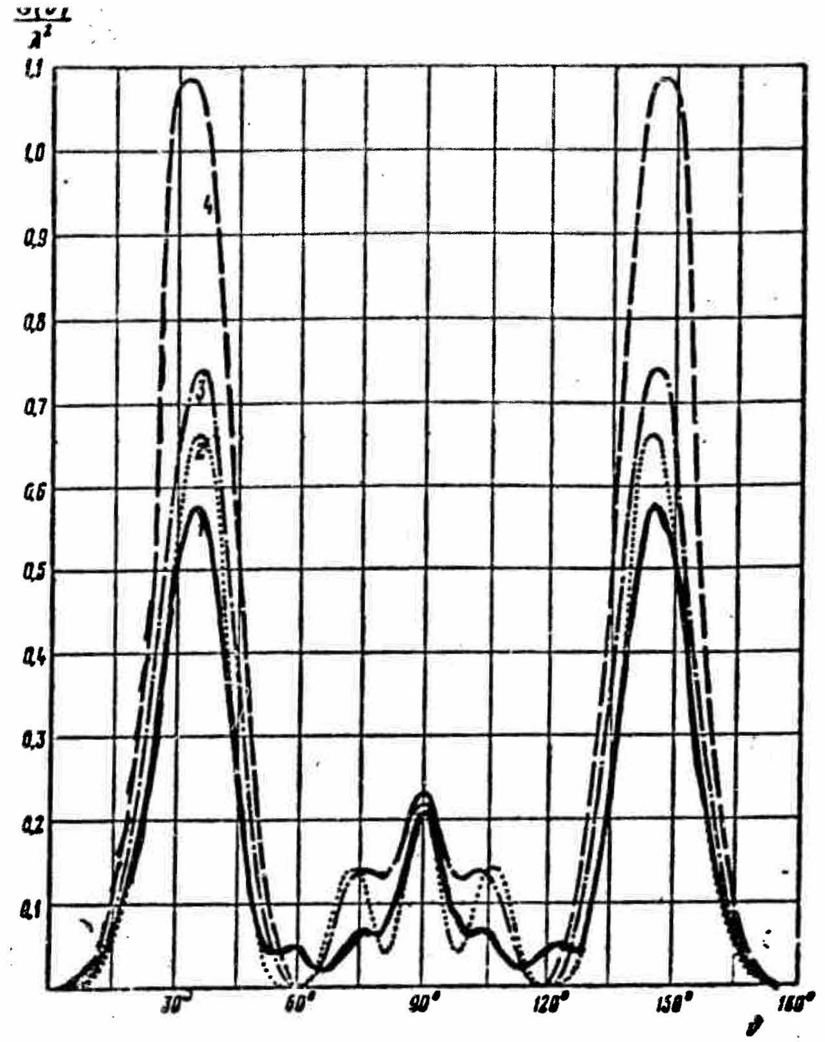

Figure 80. Diagrams for the effective scattering area of a vibrator calculated by various methods. The designations are the same as those in Figure 79:

L = 2λ

obtain the first approximation; however, its results, especially for long conductors, may depend in a substantial way on the form of the trial function. In the induced emf method [86], the current is sought in the form of a combination of trigonometric functions with unknown coefficients. These coefficients are determined by using the law of conservation of energy. This is the simplest method, but it has a number of serious defects. Thus, as a consequence of incorrectly accounting for the current component having the incident field phase, it leads to inaccurate results in the case of odd resonances (especially for long conductors), and it does not give the displacement of the resonance peaks from the values $\lambda = 2L/n$ ($n = 1, 3, 5 \ldots$) in the direction of longer wavelengths.

The results obtained by us are also approximate. However, our Equation (32.05) satisfies the reciprocity principle, and is applicable for any length vibrator. For very short vibrators $L \ll \lambda$, it changes into the asymptotic expression for the scattering characteristic of a dipole (see § 33). For vibrators with a length of several wavelengths ($L \approx n\lambda$, $n = 1, 2, 3, 4$), Equation (32.05) gives satisfactory results. Calculations performed on its basis for radar reflection with normal irradiation agree with the results of Lindroth. Good agreement is also observed with the results of Leontovich and Levin for the integral scattering characteristic. With an increase of the vibrator's length, the precision of this equation increases, and in this way it is favorably distinguished from the equations proposed for the scattering characteristics by other authors.

Moreover, the divergence between the various approximation methods indicates the necessity of performing rather detailed calculations based on precise methods, for the purpose of evaluating the actual error of the approximation methods. Such calculations may be performed, for example, by means of the method discussed in References [88, 89] or [91].

FOOTNOTE

Footnote (1) on page 177.

Let us note that one may refine Equation (28.04) by multiplying its righthand member by the factor Θ_0 (usually $\Theta_0 \approx 1$) calculated in Reference [84].

CONCLUSION

In this book, the solution of a number of diffraction problems was obtained based on the approximate consideration of the field perturbation in the vicinity of a sharp bend of the surface or a sharp edge. Equations were derived for the scattering characteristics, or in certain cases (Chapter IV), for the radar reflection thicknesses with a specified irradiation direction. The expressions which have been found have a clear physical meaning. They satisfy the reciprocity principle, and they are convenient for making calculations.

The results obtained enable us to form a more complete concept of the applicability limits of the physical optics approach. It is usually customary to assume that this approach gives reliable results only if the body's dimensions are large in comparison with the wavelength. Such an opinion is based on the following argument. The physical optics approach considers only the radiation from the uniform part of the current, and does not include in the calculations the nonuniform part of the current which is concentrated in the vicinity of the bends and the sharp edges. Therefore, when the body's dimensions are considerably larger than the wavelength, the nonuniform part of the current occupies a relatively small part of the body's surface. Therefore, one would think that its influence would be small.

But in actuality it turns out that the reliability of physical optics results depends substantially, not only on the body's dimensions, but also on the body's shape and the irradiation and observation directions. For example, with the glancing incidence of a wave on the flat face of a body, the edge zone occupied by the nonuniform part of the current is considerably broadened and the effect of this current becomes substantial. Therefore, physical optics gives qualitatively incorrect results for the field scattered by flat plates with glancing irradiation independent of the ratio between their dimensions and the wavelength. The effect of the nonuniform part of the current becomes noticeable also in those directions where, according to physical optics, the fringing field must be equal to zero or have a small value.

The problem of diffraction of a plane wave with its incidence on a cone along its axis (§ 17) serves as a clear example of how important the above-indicated factors are. Although in this case the nonuniform part of the current, concentrated near the cone's vertex, has practically no influence on the scattering, nevertheless, the physical optical approach gives values for the radar thickness which are tens of decibels smaller than the experimental values, even with large dimensions of the cone. The deciding factor here is the nonuniform part of the current flowing in the vicinity of the sharp circular base rim of the conical surface; the nonuniform part of the current has an especially large value for sharply pointed cones.

Another interesting example of a similar nature is the scattering of a plane wave by a finite paraboloid of rotation (§ 18) where the physical optics approach leads to qualitatively incorrect results. The effective scattering area calculated in this approach turns out to be a periodic function of the paraboloid length, and with certain lengths it becomes zero which most certainly does not correspond to reality.

The investigation of the diffraction of edge waves shows (Chapter V) that for flat plates one may limit oneself to consideration of secondary diffraction, if their linear dimensions are one-and-a-half to two times larger than the wavelength.

Let us note that we attempted to obtain equations for the scattering characteristics which would possess physical visualizability and which would be convenient for making calculations. In keeping with this, we were obliged to introduce various kinds of interpolation equations and simplified equations which satisfy the formulated requirements, but in return are not in the general case the dominant terms of the rigorous asymptotic expansion in powers of the small parameter λ/a.

Our purpose was not to calculate the current on the body's surface, the field in the near zone, or the integral scattering thickness. These questions are investigated in a number of other works based on the physical theory of diffraction which were already enumerated in § 25. In them, in particular, the first terms of asymptotic expansions in powers of λ/a were obtained for the integral thickness which characterizes the total power scattered by a body. However, in these works, as a rule, equations are missing for the scattering characteristics which are valid with any direction of irradiation and observation. Therefore, the results of this book and the indicated works mutually supplement one another.

At present, only a limited number of diffraction problems have yielded to theoretical studies, as a result of which experimental studies of diffraction by various bodies have taken on great importance. In Chapter VI an experimental method was discussed which enabled one to isolate in a "pure form", and to measure, the field from the nonuniform part of the current excited by a plane wave on a metal body of any shape. In the same chapter, it was shown that the well-known phenomenon of depolarization of the wave reflected from a body which is found in free space is produced by the nonuniform part of the current, or, in other words, by the surface distortion.

The investigation carried out in Chapter VII for the problem of diffraction by a thin, finite length cylindrical conductor represents a natural development and completion of the method of considering the multiple diffraction of edge waves which was applied in Chapter V. In Chapter VII equations were derived for the scattering diagram which are suitable for vibrators of an arbitrary length with any irradiation and observation directions.

The results obtained in this book show the fruitfulness of physical diffraction theory, and enable one to arrive at the solution of other more complicated problems. Such problems may be divided into two classes. Problems which may now already be solved on the basis of the known results of diffraction theory are related to the first class. As an example of such a problem, one may point to the problem of diffraction of a plane wave by a frustum of a cone or by an infinitely long cylinder with a polygonal transverse cross section. Those problems whose solution requires obtaining (using the methods of mathematical diffraction theory) a whole series of new results must be referred to the second class. In particular, in order to give a complete solution to the diffraction problem of a finite cone, it is necessary to have more precise knowledge on the diffraction laws of a semi-infinite cone.

Summing up, one may say that physical diffraction theory aids one in analyzing and sorting out the diffraction phenomena for complex bodies, poses problems for treatment by mathematical diffraction theory, and enables one to effectively apply the rigorous results of mathematical diffraction theory for the solution of new problems.

In conclusion, I express my deep thanks to L. A. Vaynshteyn for his valuable advice and regular discussion of the questions to which this book is devoted, and also for his attentive reading of the manuscript and for a number of useful remarks. I also take this opportunity to express sincere thanks to M. L. Levin for his interest in this work and his helpful remarks.

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13. ABSTRACT

The book is a monograph written as a result of research by the author. The diffraction of plane electromagnetic waves by ideally conducting bodies, the surface of which have discontinuities, is investigated in the book. The linear dimensions of the bodies are assumed to be large in comparison with the wavelength. The method developed in the book takes into account the perturbation of the field in the vicinity of the surface discontinuity and allows one to substantially refine the approximations of geometric and physical optics. Expressions are found for the fringing field in the distant zone. A numerical calculation is performed of the scattering characteristics, and a comparison is made with the results of rigorous theory and with experiments. The book is intended for physicists and radio engineers who are interested in diffraction phenomena, and also for students of advanced courses and aspirants who are specializing in antennas and the propagation of radio waves.

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